Section 7. Three-phase inverter

Recap on Three-phase square-wave inverter

Three phase bridge inverters can be viewed as extensions of the single-phase bridge circuit, as shown in figure 7.1. The switching signals for each inverter leg are displaced by 120°with respect to the adjacent legs. The output line-line voltages are determined by the potential differences between the output terminals of each leg. Symmetrical three phase voltages across a three-phase load can be produced by switching the devices ON for either 180° or 120° of the output voltage waveform. With 180° conduction, the switching sequence is T1T2T3 – T2T3T4 – T3T4T5 – T4T5T6 – T5T6T1 – T6T1T2 – T1T2T3 - .... for the positive A-B-C phase sequence and the other way round for the negative (A-C-B) phase sequence. With 120° conduction, the switching pattern is T1T2 – T2T3 – T3T4 – T4T5 – T5T6 – T6T1 – T1T2 for the positive A-B-C sequence and the other way round for the negative (A-C-B) phase sequence.

Whenever an upper switch in an inverter leg connected with the positive DC rail is turned ON, the output terminal of the leg goes to potential $+V_d/2$ with respect to the center-tap of the DC supply. Whenever a lower switch in an inverter leg connected with the negative DC rail is turned ON, the output terminal of that leg goes to potential $-V_d/2$ with respect to the center-tap of the DC

supply. Note that a center-tap of the DC supply V_d has been created by connecting two equal valued capacitors across it. The center-tap is assumed to be at zero or earth potential. However, this contraption is artificial and really not essential; the center-tap may not exist in practice. We assume**,** henceforth, that the three-phase load connected to the output terminals of the inverter is balanced.

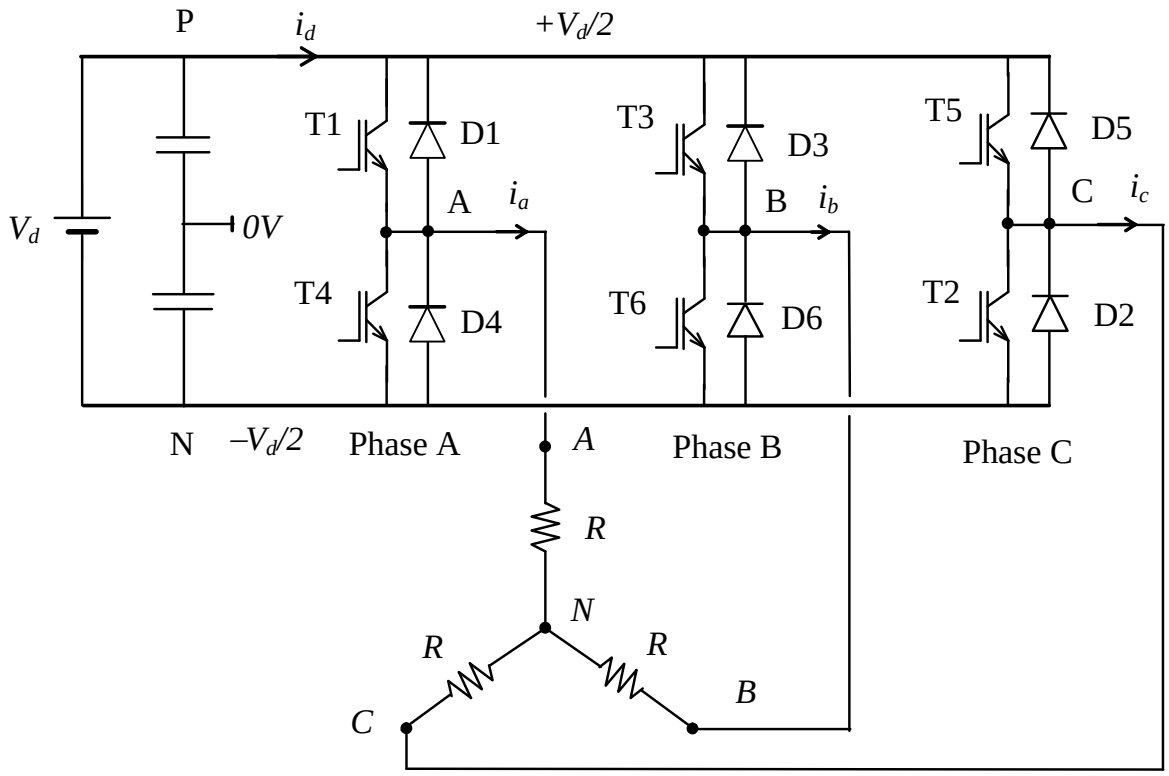


Figure 7.1

Inverter waveforms with 180° conduction angle

In this case, each switch is turned ON for 180°. Switches T1 and T4, which belong to the left-most inverter leg, produces the output voltage for phase A. The switching signals for T1 and T4 are complementary, as are for T3 and T6 or T5 and T2. The switching signals for switches T3 and T6, (which are for phase B, belonging to the middle leg), are delayed by 120° from those for T1 and T4 respectively, for the ABC phase sequence. Similarly, for the same phase sequence, the switching signals for switches T5 and T2 are delayed from the switching signals for T3 and T6 by 120°. The phase terminal voltages at A, B and C (sometimes called respective pole voltages) are determined by the states of the switches connected at each pole. Note that with 180° conduction (i.e., complementary switching), each pole voltage can have only two values (or discrete states), namely $+\frac{V_d}{2}$ or $-\frac{V_d}{2}$. Considering that there are three poles, the number possible output voltage states from the inverter are $2^3 = 8$.

Line-line voltage waveforms

The line-line voltages, v_{AB}, v_{BC} and v_{CA} are determined from the switching states at the poles) and the DC source voltage, (V_d). Thus, when switches T1 and T3 are ON, v_{AB} = 0V, when T1 and T6 are ON, $v_{AB} = +V_d$, and so on. The line-line voltages v_{AB}, v_{BC} and v_{CA} (for the +ve or ABC phase sequence) are therefore quasi-square waveforms of

120° of ON and 60° of OFF durations, as shown in figure 7.2. Each is phase displaced from its adjacent ones by 120°.

Line-neutral voltage waveforms

Line-neutral voltages are determined from the switching states and the neutral point voltage of the load which can be found by assuming that the load consists of a balanced three-phase resistor bank. For instance, if T1, T3 and T2 are ON, the potential of the neutral point of the load is $\frac{2}{3}V_d$ and therefore V_{AN} and V_{BN} will each be at potentials $\frac{1}{3}V_d$ while v_{CN} will be at $-\frac{2}{3}V_d$. Similarly, when T4, T2 and T3 are ON, the potential of the neutral point will be $\frac{1}{3}V_d$. As a result, the potential v_{BN} will become $\frac{2}{3}V_d$ and v_{AN} and v_{CN} will each be at $-\frac{1}{3}V_d$.

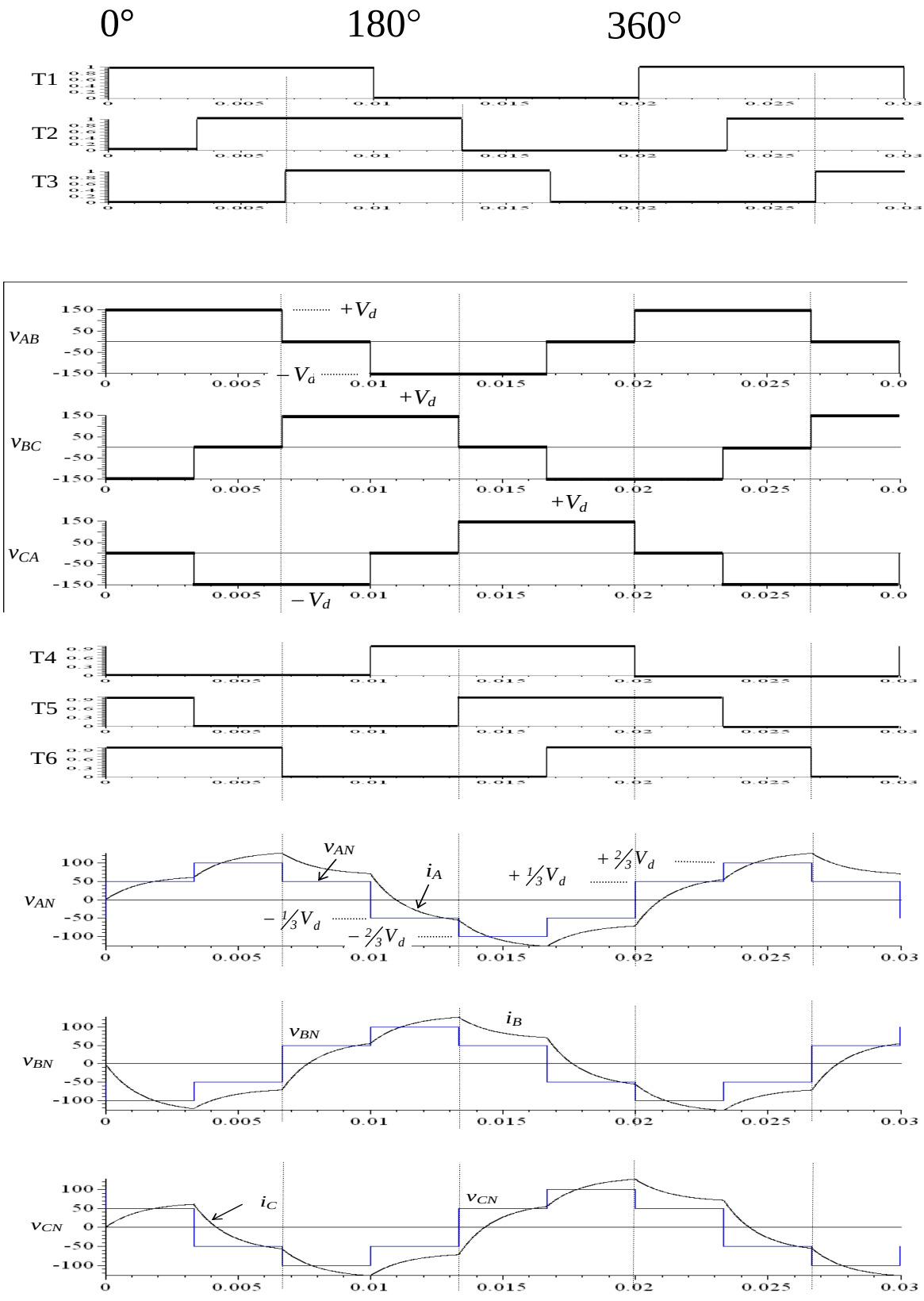
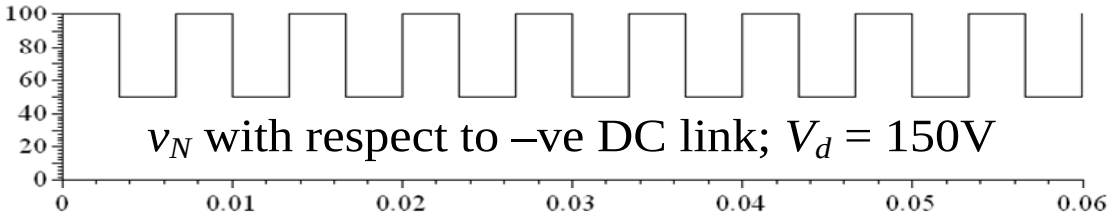
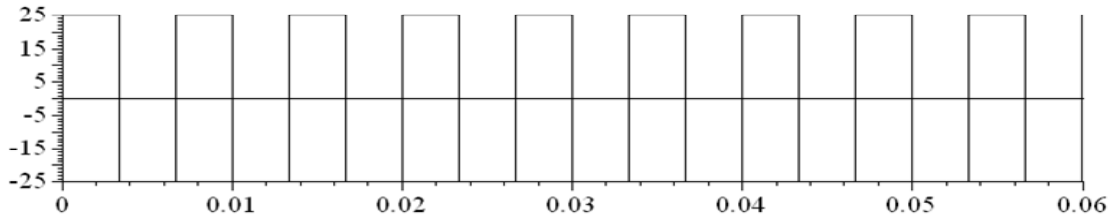

0°
180°
360°
T1
T2
T3
v_{AB}
v_{BC}
v_{CA}
$+V_d$
$-V_d$
T4
T5
T6
v_{AN}
i_A
$+\frac{1}{3}V_d$
$+\frac{2}{3}V_d$
$-\frac{1}{3}V_d$
$-\frac{2}{3}V_d$
v_{BN}
i_B
v_{CN}
i_C

Figure 7.2



v_N with respect to –ve DC link; V_d = 150V



v_N with respect to center-tap of DC link; V_d = 150V

Figure 7.3

Analysis of output voltage

Line-line voltage

The line-line output voltages are obtained by subtracting two square-wave waveforms which are 120° displaced from each other. Each of these waveforms would consist of harmonics orders 1, 3, 5, 7, 9, … and so on. Because of the 120° phase shift between the waveforms, the triplen harmonics (of order which are multiples of 3) of both will of the same phase and hence these cancel in the process of subtraction. Consequently, the triplen order harmonic voltages are eliminated from the line – line voltage. The remaining harmonics are at $n = 6r \pm 1$ where r is any positive integer, the n^{th} harmonic having an amplitude 1/n times the fundamental component.

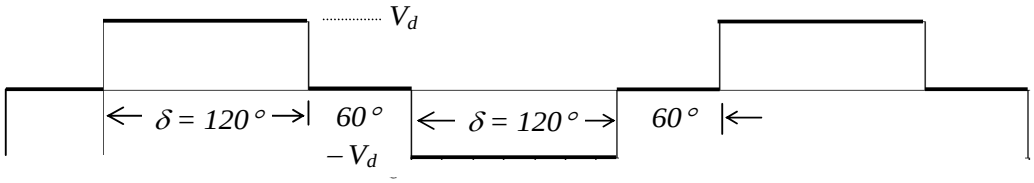


Figure 7.4

The line-line quasi-square output voltage waveform of figure 7.4 has amplitude V_d and duration δ = 120°. Fourier series representation of this waveform is

$$v_{l-l} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_d}{n\pi} \sin\left(\frac{n\delta}{2}\right) \cos n\omega_o t \qquad (7.2)$$

$$= \frac{2\sqrt{3}}{\pi} V_d \left[\cos(\omega_o t) + \frac{1}{5}\cos(5\omega_o t) + \frac{1}{7}\cos(7\omega_o t) + \frac{1}{11}\cos(11\omega_o t) + \ldots\ldots \right] \qquad (7.3)$$

The RMS values of the fundamental and higher order output voltages are,

$$V_{l-l,1} = \frac{\sqrt{6}\,V_d}{\pi}\ ; \quad V_{l-l,5} = \frac{\sqrt{6}\,V_d}{5\pi}\ ; \quad V_{l-l,7} = \frac{\sqrt{6}\,V_d}{7\pi}\ ; \quad V_{l-l,11} = \frac{\sqrt{6}\,V_d}{11\pi}\ ; \qquad (7.4)$$

and so on.

Thus, the fundamental RMS output, $V_{l-l,1} = 0.78V_d$

and $V_{l-l,h} = \frac{0.78V_d}{h}$ where $h = 6n \pm 1$ and $n = 1, 2, 3, \ldots..$

(7.5)

Line-neutral voltage

The line-neutral voltage waveform for this inverter is as shown in figure 7.5. Fourier series representation of this waveform is given by

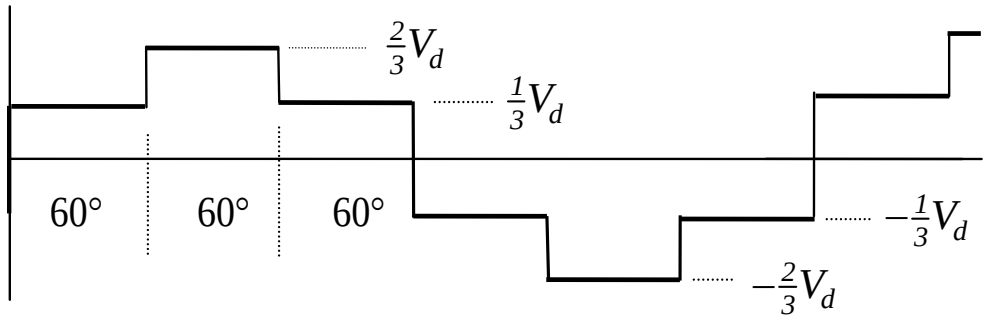


Figure 7.5

$$v_{l-n} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_d}{n\pi} \sin\frac{n\delta}{2} \cos\left(n\omega_o t\right) \qquad (7.6)$$

$$= \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{4}{\pi}\frac{V_d}{3n} \sin\frac{n180^\circ}{2} + \frac{4}{\pi}\frac{V_d}{3n} \sin\frac{n60^\circ}{2} \right] \cos\left(n\omega_o t\right) \tag{7.7}$$

$$= \frac{2}{\pi}.V_d\left[\cos\omega_o t + \frac{1}{5}\cos 5\omega_o t + \frac{1}{7}\cos 7\omega_o t + \frac{1}{11}\cos 11\omega_o t + \cdots\right] \tag{7.8}$$

RMS values of the fundamental and higher order terms of the line-neutral voltage are:

$$V_{l\text{–}n,1} = \frac{\sqrt{2}V_d}{\pi}\ ;\quad V_{l\text{–}n,5} = \frac{\sqrt{2}V_d}{5\pi}\ ;\quad V_{l\text{–}n,7} = \frac{\sqrt{2}V_d}{7\pi}\ ;\quad V_{l\text{–}n,11} = \frac{\sqrt{2}V_d}{11\pi}\ ; \tag{7.9}$$

and so on.

Three-phase SPWM Inverters

Output voltage control of the three-phase inverter

Available methods are:

- Input DC voltage regulation - not suitable for voltage source inverters except for slow adjustment of output voltage.

- SPWM – the method now widely used for low to medium power applications.

Unlike the case of a single-phase inverter, variable phase displacement between inverter legs cannot be used as a means for output voltage variation. This is due to the restriction that phase displacement of $2\pi/3$ (120°) between phases must be maintained in order to obtain balanced three-phase output voltage.

Three-phase SPWM Inverter

Three-phase SPWM inverters are controlled in the same way as a single-phase SPWM inverter. Three sinusoidal modulating signals at the frequency of the desired output but displaced from each other by 120° are compared with a triangular carrier waveform of suitably high frequency. The resulting switching signals from each comparator are used to drive the inverter switches of the corresponding leg. The switching signals for each inverter leg are complementary, and the switching signals for each switch has 180° of conduction. These are shown in figure 7.2 for the inverter of figure 7.1.

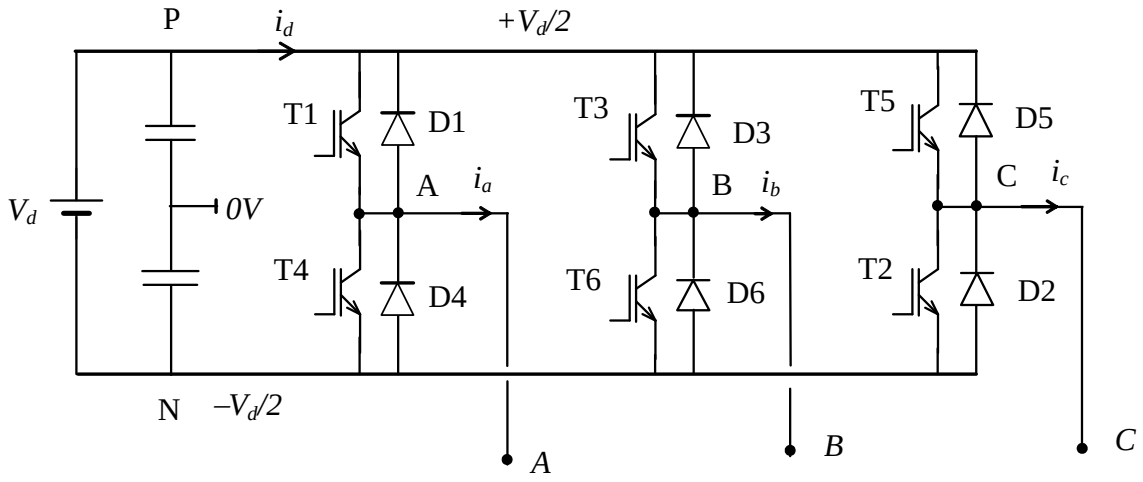
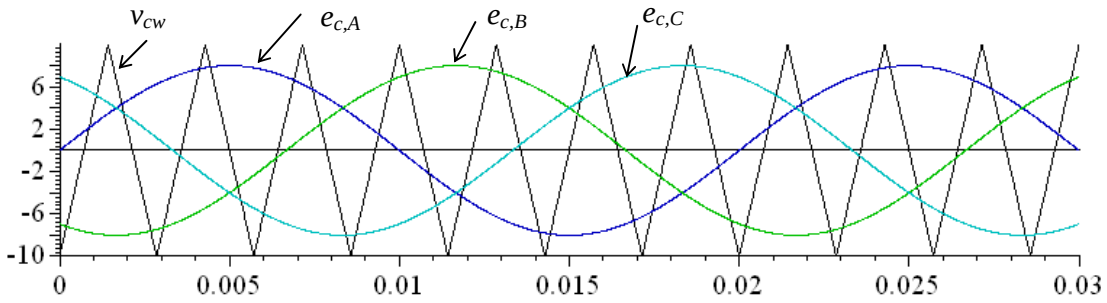
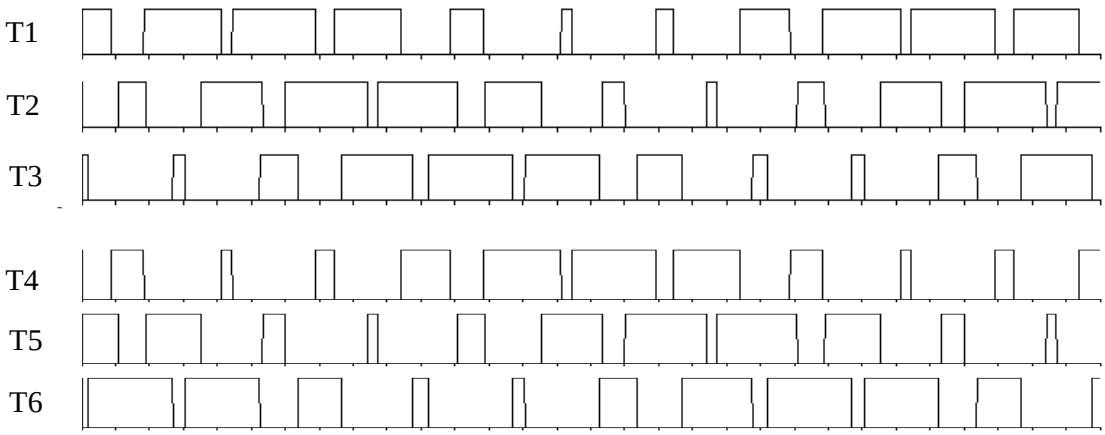


Figure 7.6


v_{cw}
$e_{c,A}$
$e_{c,B}$
$e_{c,C}$
6
2
-2
-6
-10
0
0.005
0.01
0.015
0.02
0.025
0.03


T1
T2
T3
T4
T5
T6

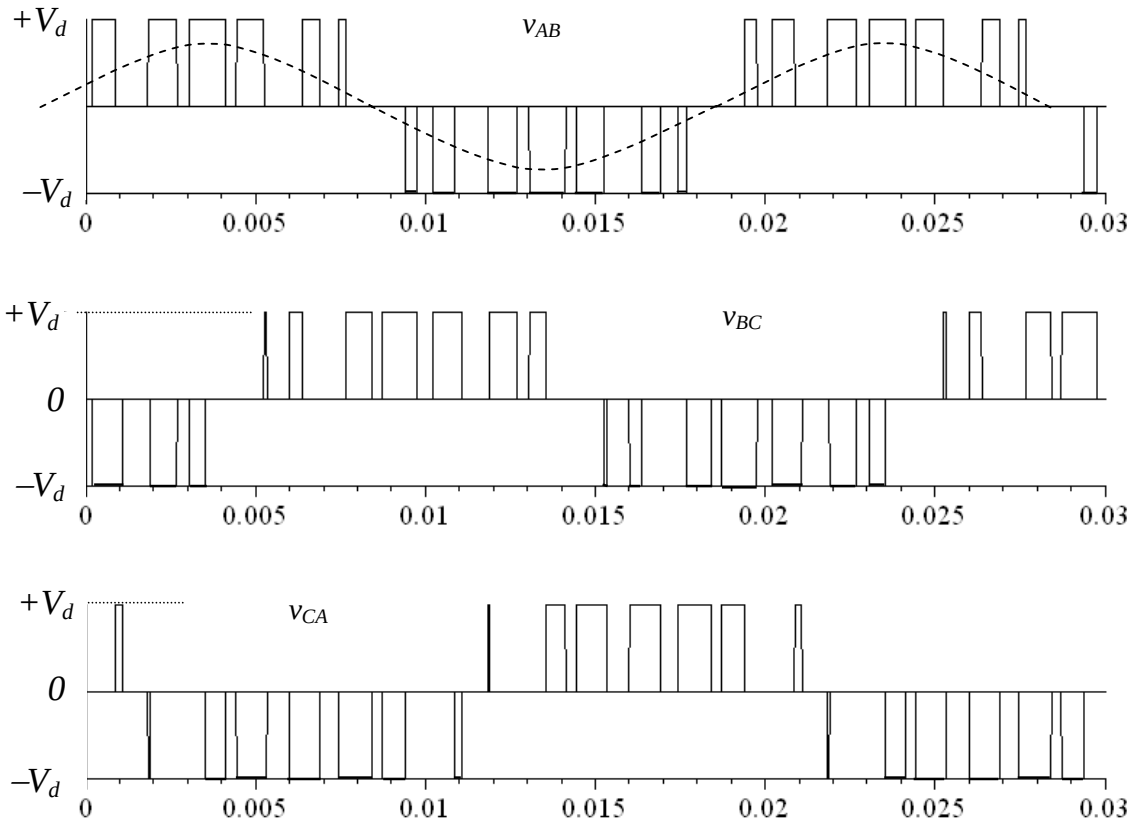

+Vd
vAB
–Vd
0
0.005
0.01
0.015
0.02
0.025
0.03
+Vd
vBC
0
–Vd
0
0.005
0.01
0.015
0.02
0.025
0.03
+Vd
vCA
0
–Vd
0
0.005
0.01
0.015
0.02
0.025
0.03

Figure 7.7

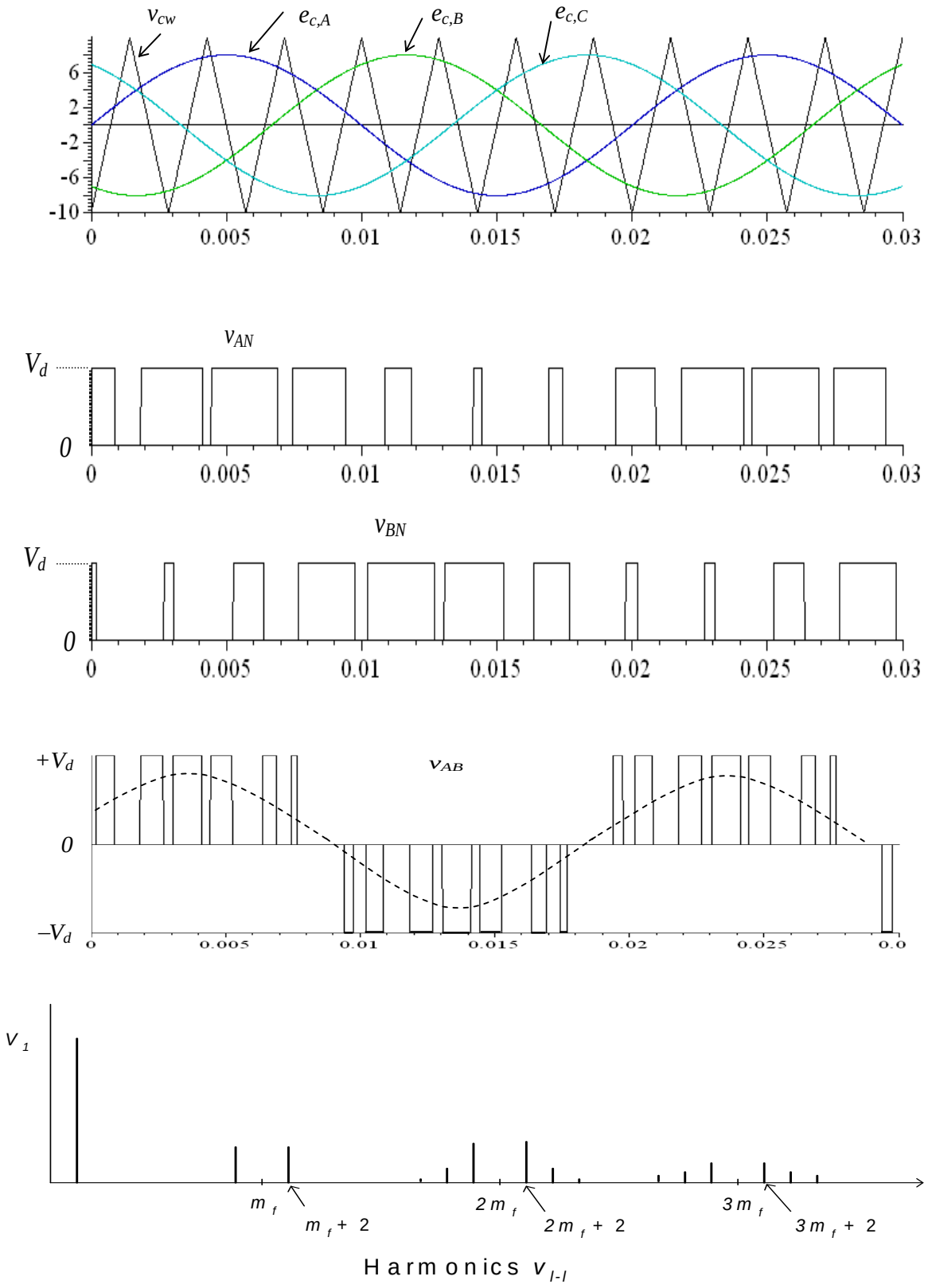

v_{cw}
$e_{c,A}$
$e_{c,B}$
$e_{c,C}$
6
2
-2
-6
-10
0
0.005
0.01
0.015
0.02
0.025
0.03
v_{AN}
V_d
0
v_{BN}
$+V_d$
v_{AB}
$-V_d$
V_1
m_f
$m_f + 2$
$2m_f$
$2m_f + 2$
$3m_f$
$3m_f + 2$
Harmonics v_{l-l}

Figure 7.8

It should be noted from the above waveforms in figure 7.2-3 that an identical amount of DC voltage exists in each line-neutral voltage v_{AN} and v_{BN} when these are measured with respect to the negative DC link voltage bus. The DC components are cancelled when v_{AB} is obtained by subtracting v_{BN} from v_{AN}. It should also be noted that the v_{AB} waveform is 30° ahead of the control voltage (e_{cA}) for phase A.

The considerations for selecting the triangular carrier frequency and their synchronization with the modulating waveforms are similar to what have been described for single-phase SPWM inverters. Thus, m_f should be an odd integer which is also a multiple of three when it is less than 21 and that the slopes of the modulating waveforms and the carrier waveforms at the zero crossings of the modulating waveforms should be opposite.

When frequency variation in addition to voltage variation is required, the above restrictions should be maintained at all frequencies. In order to improve the inverter efficiency, it is desirable to keep m_f constant when the output frequency is low, otherwise, too many switchings would occur unnecessarily.

Linear Modulation Range, m < 1

Considering that the positive DC bus voltage is $+V_d/2$ and the negative DC bus voltage is $-V_d/2$ with respect to the center-tap of the DC supply, the output voltage waveform of a phase leg is a pulsewidth modulated bipolar AC waveform of magnitude $\frac{V_d}{2}$. The RMS value of the fundamental of this voltage varies linearly with the depth of modulation *m*. Thus,

$$V_{An,1} = m \cdot \frac{V_d}{2\sqrt{2}} = 0.354m \cdot V_d \qquad (7.1)$$

where *m* is the depth of modulation. This has been indicated as the line-neutral voltage because, with SPWM and balanced three-phase load, the potential of the load neutral point and that of the DC supply center-tap should be the same. The RMS value of the fundamental line-line voltage is

$$V_{AB,1} = m \cdot \frac{\sqrt{3}V_d}{2\sqrt{2}} = 0.612m \cdot V_d \qquad (7.2)$$

The calculation of harmonics in the output of a three phase inverter is rather involved. It is best managed on a computer. Figure 7.3 and Table I shows the relative harmonic amplitudes of line-line RMS voltages for a large and odd m_f.

Table I (Source: N. Mohan et al, Power Electronics)

m / *n*	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
1	0.122	0.245	0.367	0.490	0.612
$m_f \pm 2$	0.010	0.037	0.080	0.135	0.195
$m_f \pm 4$				0.005	0.011
$2m_f \pm 1$	0.116	0.200	0.227	0.192	0.111
$2m_f \pm 5$				0.008	0.020
$3m_f \pm 2$	0.027	0.085	0.124	0.108	0.038
$3m_f \pm 4$		0.007	0.029	0.064	0.096
$4m_f \pm 1$	0.100	0.096	0.005	0.064	0.042
$4m_f \pm 5$			0.021	0.051	0.073
$4m_f \pm 7$				0.010	0.030

Over-modulation, m > 1

If the peak amplitude of the control voltage is greater than the peak of the carrier waveform, the fundamental output voltage will increase, eventually becoming $0.78V_d$ when *m* is infinite and the line-line output waveform becomes a square-wave. Over-modulation is thus a means of

increasing the output voltage range of a SPWM inverter. When over-modulation is used, more lower- order and sideband harmonics and their multiples will exist around m_f. However, the dominant harmonics will not be as large in amplitude as with operation in the linear range.

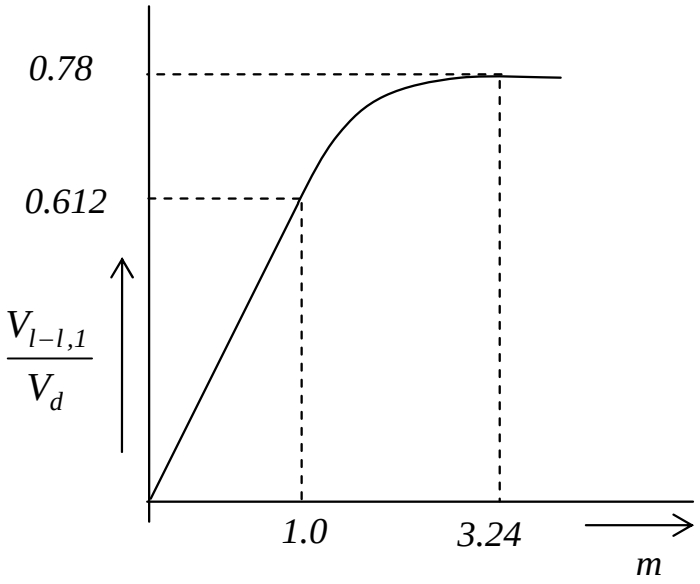


Figure 7.9

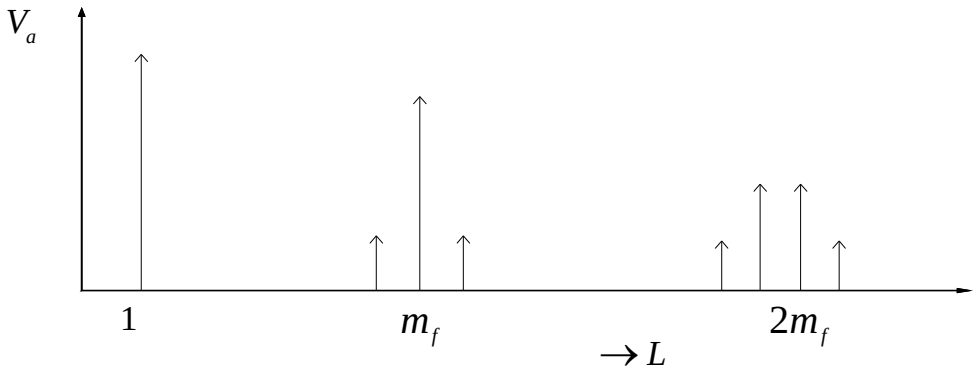
Modulation Schemes: Sinusoidal PWM

$$v_{an} = \frac{V_d}{2} \frac{V_{refa}}{\hat{V}_{tri}}$$

This assumes that v_{refa} varies little over a switching period. This is true when m_f is high ($\geq$ 21). Also, it is assumed that m_f is odd and v_{ref} for all three phase are synchronised v_{tri} with opposite slopes at each zero

crossing. Thus m_f is assumed to be an odd integer. If these assumptions are met, (i.e., with Synchronous PWM) the harmonic spectrum will only have component at multiples of the carrier and their sideband frequencies given by

$$f_h = (jm_f \pm k) f_1$$



SPWM with Overmodulation

When $m > \hat{V}_{tri}$, the output voltage spectrum will contain lower order harmonics including sub-harmonics (harmonics of lower frequency than the modulating frequency). Under such circumstances, the above assumptions should be met in order to minimise these hard-to-filter-out harmonics.

For $m \leq 1$

$$v_a = m\frac{V_d}{2}$$

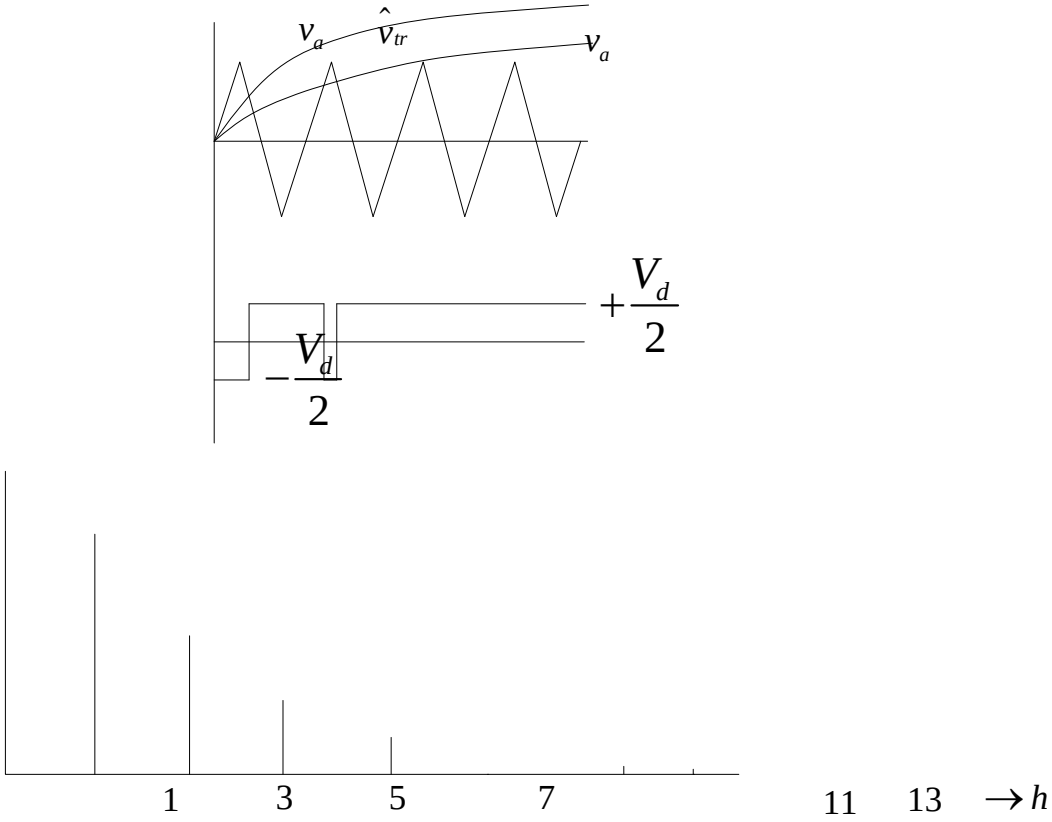
$$\Rightarrow V_a = m\frac{V_d}{2\sqrt{2}} \text{ in RMS}$$

$$V_{ll} = \frac{\sqrt{3}mV_d}{2\sqrt{2}} \text{ in RMS}$$

$$V_{ll} = 0.612mV_d$$

For $m > 1$

$V_{ll} \to 0.78V_d$ when $m \to 3.24$

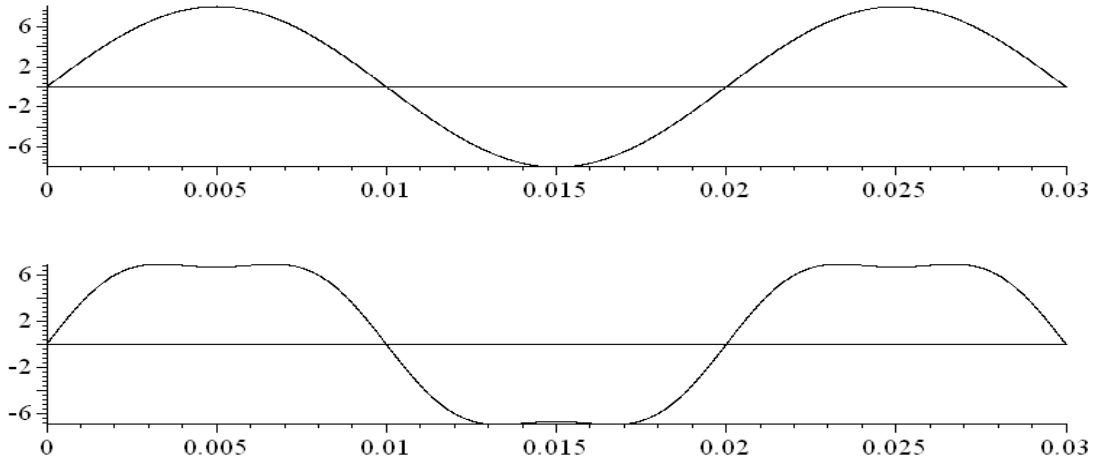


SPWM with 3rd Harmonic Injection

The amplitude of the fundamental can be increased further by adding a third harmonic to the modulating waveform as indicated in the figure below. If the modulating waveform is

$$e_{c,A} = m \times sin(\omega_o t) + \frac{1}{6} \times m \times sin(3\omega_o t)$$

for $m \leq 1$.



Figure

It can be shown that the fundamental line-line output voltage can be raised to 1.155 of what is obtained with only the fundamental frequency modulating waveform. Although some third harmonic is added to the modulating

waveform, the third harmonic phase currents in a star connected load will always cancel.

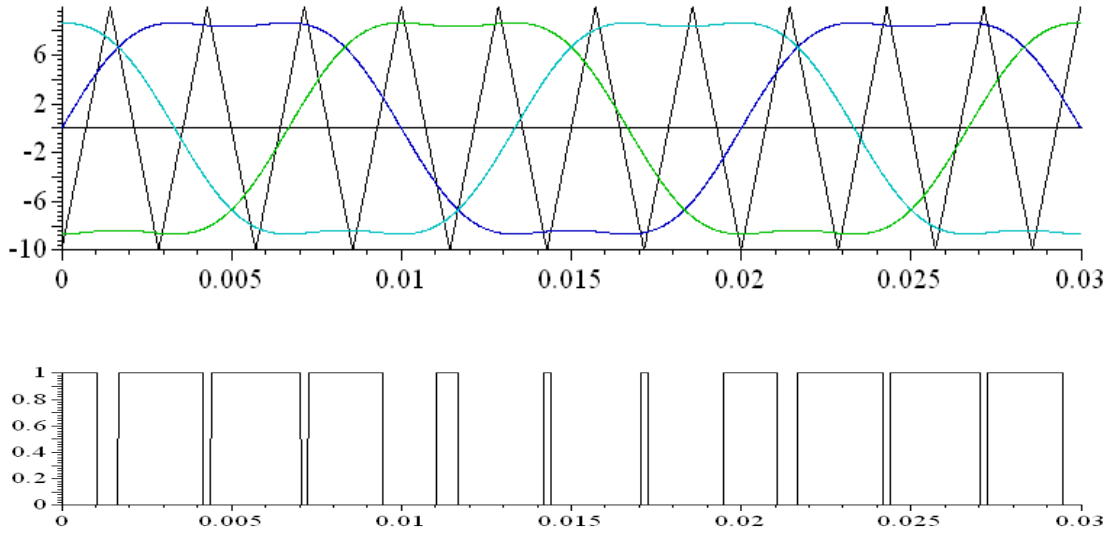


Figure. PWM switching signal for phase leg A.

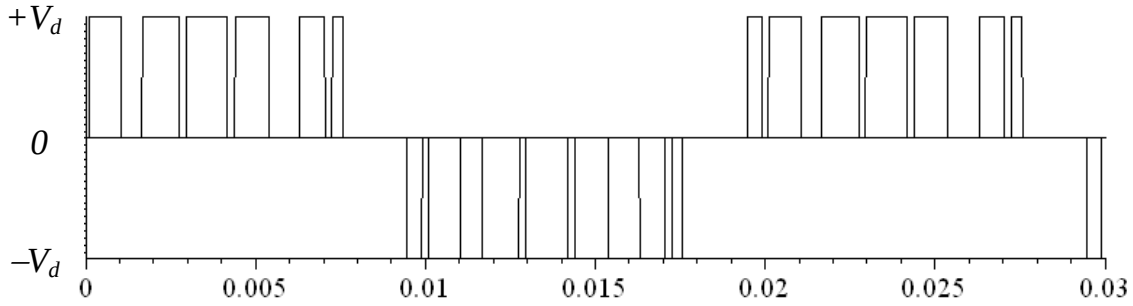


Figure. For m = 1, and 1/6th of third harmonic injection

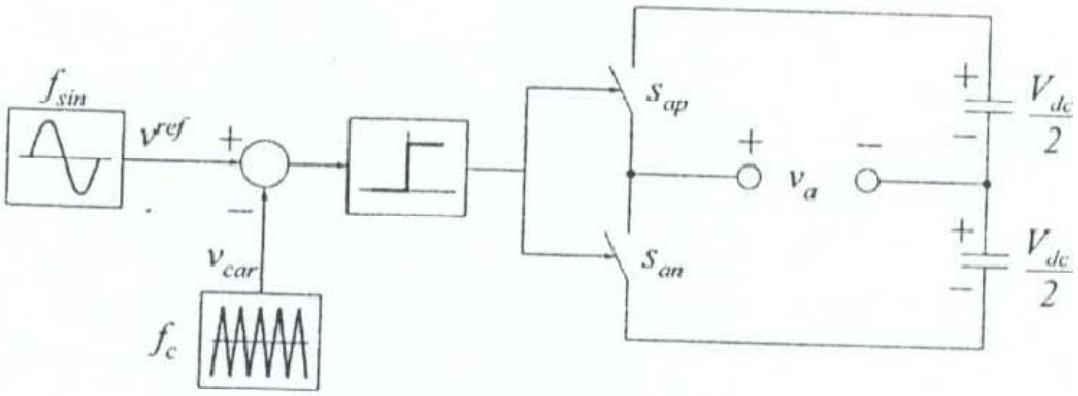
Note from figure above that even with m = 1 and 16.67% of third harmonic injection in the reference voltages, the amplitude of the modulating waveform does not exceed the amplitude of the career. Additional low-order harmonics that are present in v_{AB} are thus not as much as

they would be if *m* was increased beyond 1 to increase the output voltage.

Note that pulse dropping does not occur.

PWM pulse generation

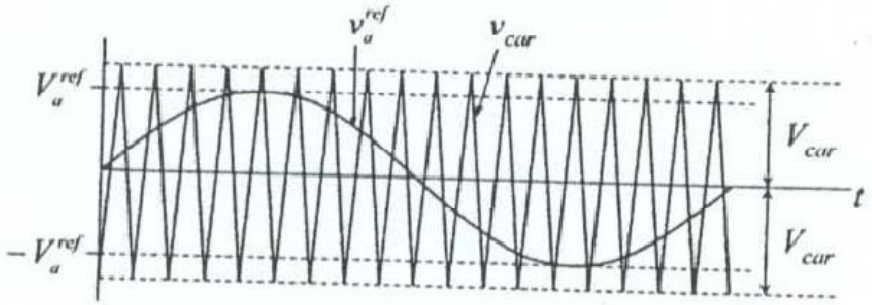
A. Analogue (SPWM Asynchronous)

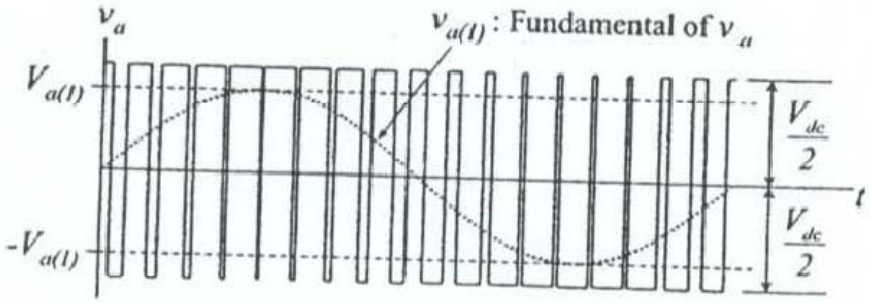


- Determine the switching state by magnitude of v^{ref} and v_{car}

  If $|v^{ref}| > |v_{car}|$, then s_{ap} on,
  If $|v^{ref}| < |v_{car}|$, then s_{an} on

- Switching frequency is the same as carrier wave *frequency*, f_c





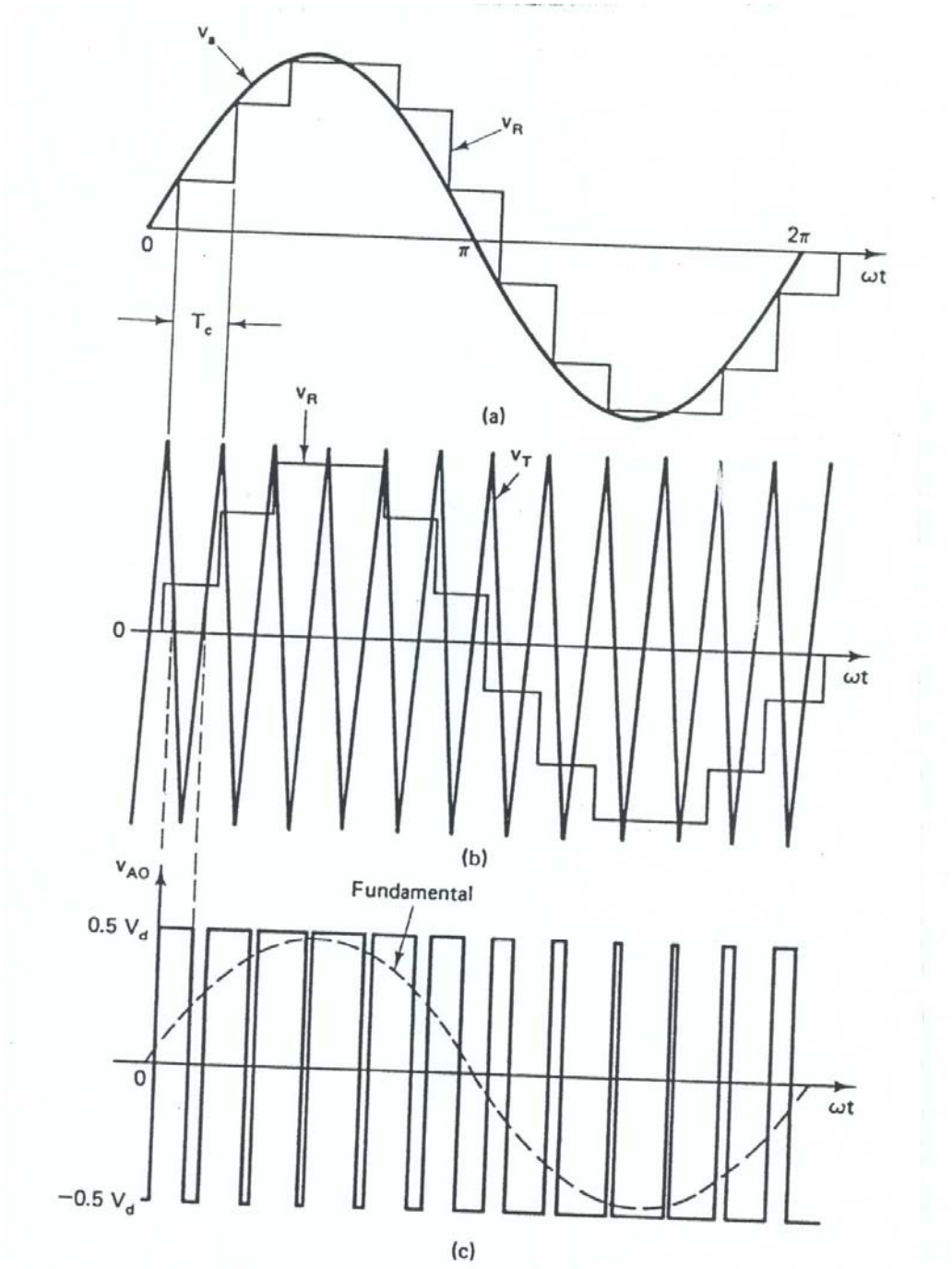
$$\frac{V_a^{ref}}{V_{car}} = \frac{V_{a(1)}}{\left(\frac{V_{dc}}{2}\right)} = \frac{2 \cdot V_{a(1)}}{V_{dc}} \Rightarrow V_{a(1)} = \frac{V_{dc}}{2} \cdot \frac{V_a^{ref}}{V_{car}}$$

$v_a > v_{tri}$ T_1 on, T_4 off

$v_a < v_{tri}$ T_1 off, T_4 on

When the modulating waveforms v_a, v_b and v_c have noise, the comparators may produce the wrong outputs

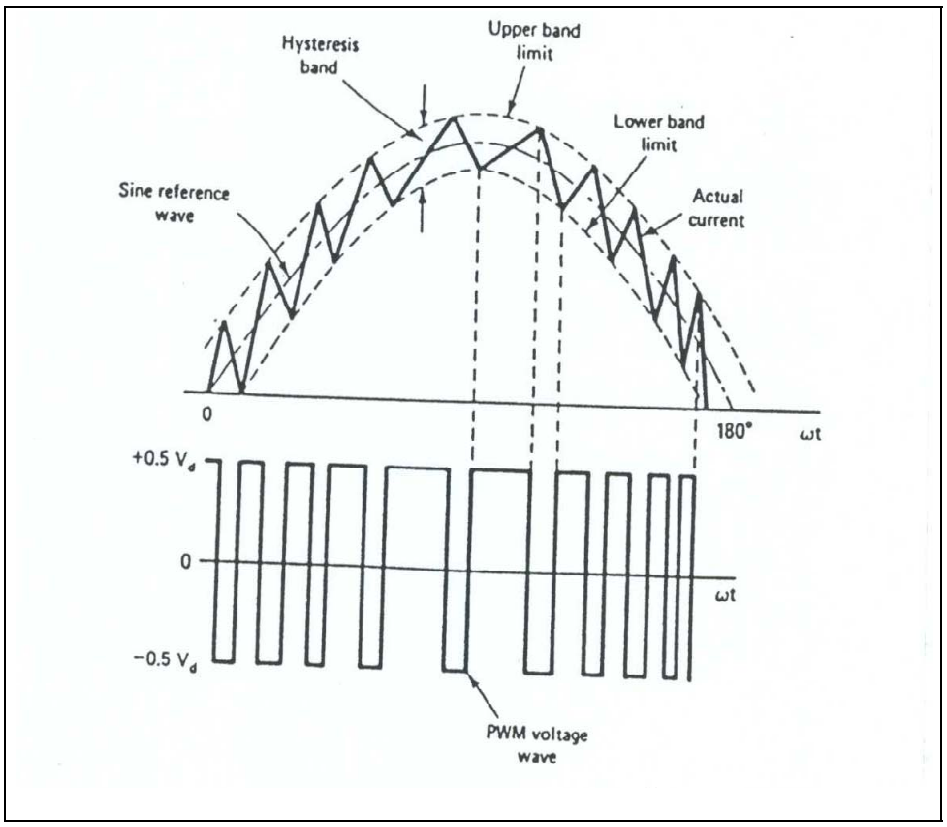
B. Synchronous or Regular Sampled PWM: also called uniform or regular sampled. Discrete method: the modulating waveform is sampled at regular intervals, only at the peak of v_{tri}.



SPWM with Uniform (regular) Sampling

This method eliminates the low order harmonics from the output voltage. Only components at the carrier and its sideband frequencies may occur.

(C) Carrier-less: Hysteresis Band PWM



- Normally used for supplying sinusoidal currents rather than voltage, to a 3-phase load.
- Current ripple is kept within limit.
- The carrier frequency can be variable

(d) Adaptive hysteresis band

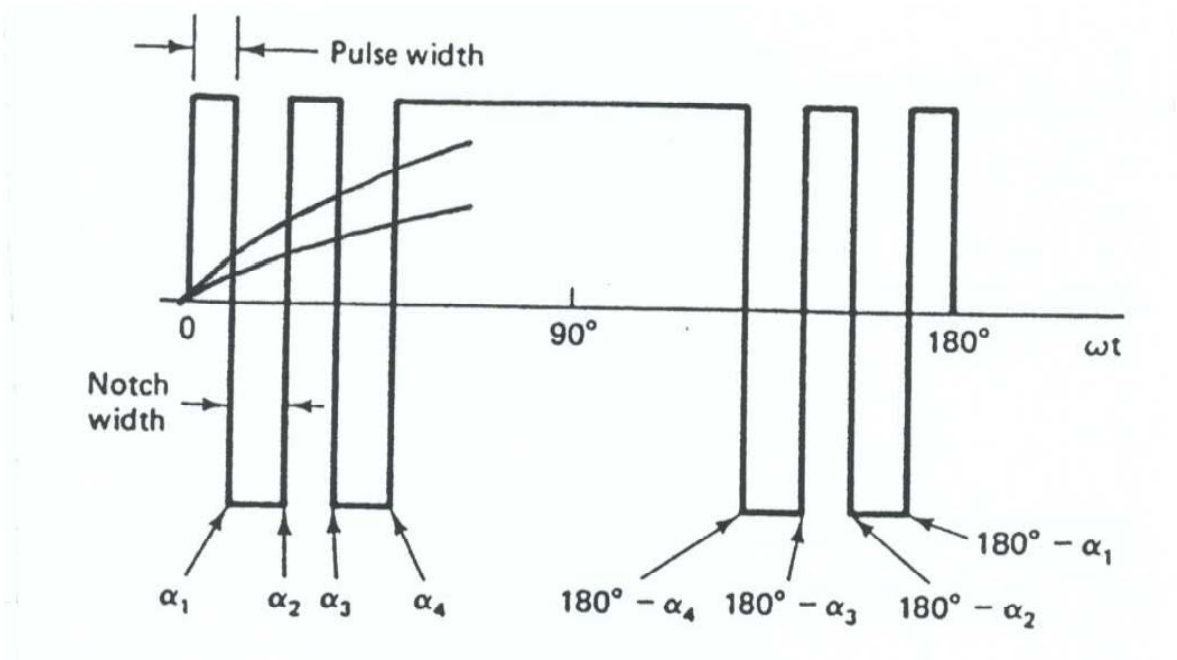
The carrier frequency is kept nearly constant by adjustment of the hysteresis band continuously. The hysteresis band is now programmable – it is adjusted such that the switching frequency remains nearly constant. Volt-second balance in each PWM switching period is employed.

The adjustment of the hysteresis band can be determined by the DC voltage of the inverter (feed forward) and by the level of current ripple in the output terminals of the inverter.

(e) Selective Harmonic Elimination

The switching angles of an inverter can be so selected that certain low-order harmonics are eliminated from the output waveform. For example, the 5^{th}, 7^{th}, 11^{th} and 13^{th} harmonics, which are the most offending harmonics of a three-phase, six-step quasi-square inverter, may be eliminated by selecting the switching angles α_1 - α_4.

Waveform of two-level SHE PWM are N pulses/half cycle of the inverter, N being an odd number. It is assumed that the inverter output has quarter-wave symmetry



$$\alpha_k = \pi - \alpha_{N-k+1} \text{ for } k=1,2,\ldots N$$

$$B_n = \frac{4E}{n\pi}(-1)^N \left\{1 + 2\sum_{k=1}^{N}(-1)^K \cos n\alpha_K\right\} \text{ for } N = \text{ odd}$$

$$B_n = 0 \text{ for N = even.}$$

It can be shown that *2N-1* (i.e., *3,5*, …) odd harmonics can be eliminated from the output. The equation for B_n can be solved with the following constrains

$$B_1(\alpha) = mE$$

$$B_k(\alpha) = 0$$

The resulting equations are transcendental without closed form solution. An iterative process of solution must be employed. For instance, with *N=3*,

$$B_n = \frac{4\mathrm{E}}{\mathrm{n}\pi}(1 - \cos n\alpha_1 + \cos n\alpha_2)$$

For $n = 3$ and 5,

$$1 - \cos 3\alpha_1 + \cos 3\alpha_2 = 0$$

$$1 - \cos 5\alpha_1 - \cos 5\alpha_2 = 0$$

Solving these two equations, α_1 and α_2 can be found. These will eliminate harmonics of order 3 and 5 of a single-phase inverter.

(f) Space vector Modulation (SVM)

The SVM method retains the advantage of 16% third harmonic injection of SPWM, while minimising the number of switching states of the inverter. It thus utilized the DC link voltage better than SPWM, has fixed switching frequency and low switching loss.

Any three-phase sinusoidal balanced voltage set can be described in a complex ($\alpha\beta$) plane. The relationships are given below (using Park's transformation)

The space vector $\overline{v}$ can be projected on to the orthogonal frame $\alpha - \beta$ and find V_{ab}, V_{bc} and V_{ca} that will lead to $\overline{v}$. Note that a balance three-phase voltage of arbitrary amplitude can also be represented by the voltage vector $\overline{v}$ given by

$$\overline{v} = V_a + aV_b + a^2V_c$$

where V_a, V_b and V_c are the line-neutral voltages.

In order to produce a certain voltage vector $\overline{v}$, the required line-line voltages V_{ab}, V_{bc} and V_{ca} can be synthesised by PWM control of a pair of switches or by modulation of voltage vectors that are applied to the load. The inverter can eight produce voltage vectors, according to the states of the three switches, from 000 to 111, as indicated in the table below.

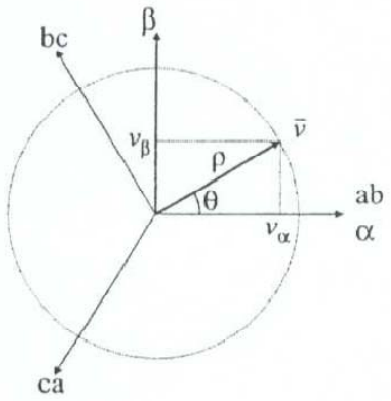
$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} \quad \text{where} \quad T_{\alpha\beta/abc} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

- Space vector

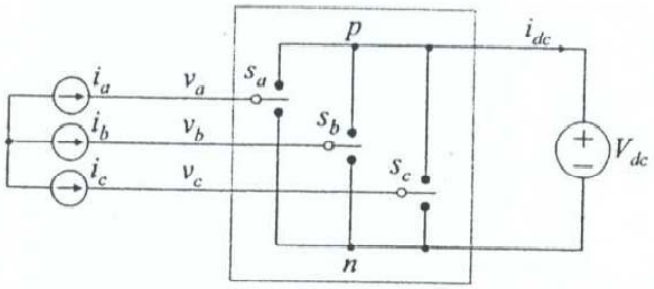
$$\bar{v} = \rho \cdot e^{j\theta}$$

$$\rho = \sqrt{v_\alpha^2 + v_\beta^2}$$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right)$$



☞ If V_m is the amplitude of balanced, symmetrical, three-phase line-to-line voltages, then $\rho = \sqrt{\frac{3}{2}} \cdot V_m$



s_a	s_b	s_c	Switching state	i_{dc}	v_{ab}	v_{bc}	v_{ca}
0	0	0	nnn	0	0	0	0
0	0	1	nnp	i_c	0	$-V_{dc}$	V_{dc}
0	1	0	npn	i_b	$-V_{dc}$	V_{dc}	0
0	1	1	npp	i_b+i_c	$-V_{dc}$	0	V_{dc}
1	0	0	pnn	i_a	V_{dc}	0	$-V_{dc}$
1	0	1	pnp	i_a+i_c	V_{dc}	$-V_{dc}$	0
1	1	0	ppn	i_a+i_b	0	V_{dc}	V_{dc}
1	1	1	ppp	$i_a+i_b+i_c$	0	0	0

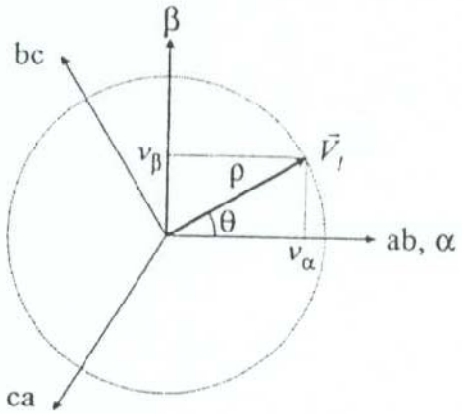
For example, when the inverter output voltage vector is 100 (or V_{pnn})

$$\bar{V}_{pnn} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{pnn} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{pnn} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} V_{dc} \\ 0 \\ -V_{dc} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{3}{2}} \cdot V_{dc} \\ \sqrt{\frac{1}{2}} \cdot V_{dc} \end{bmatrix}$$

$$\bar{V}_{pnn} = \bar{V}_l = \rho \cdot e^{j\theta}$$

$$\rho = \sqrt{2} \cdot V_{dc}$$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right) = 30°$$



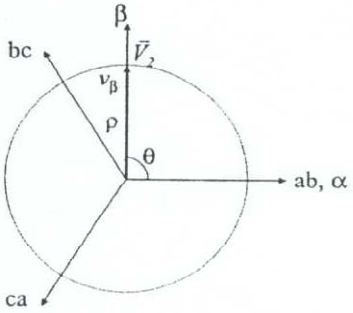
The inverter output voltage vectors can be represented in hexagonal plane as indicated below with the help of six 60° sectors.

$$\vec{V}_{ppn} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{ppn} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{ppn} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ V_{dc} \\ -V_{dc} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \cdot V_{dc} \end{bmatrix}$$

$$\vec{V}_{ppn} = \vec{V}_2 = \rho \cdot e^{j\theta}$$

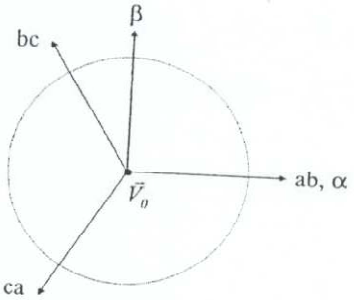
$$\rho = \sqrt{2} \cdot V_{dc}$$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right) = 90°$$

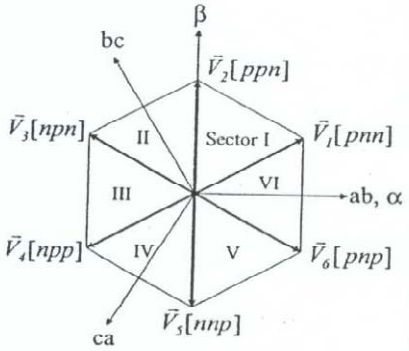


$$\vec{V}_{ppp} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{ppp} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{ppp} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

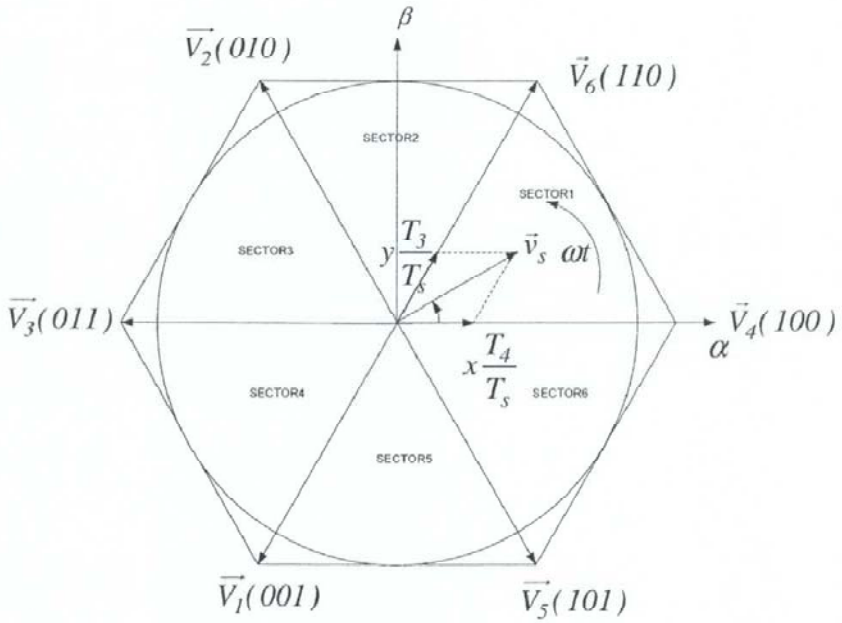
$$\vec{V}_{ppp} = \vec{V}_0 = 0$$

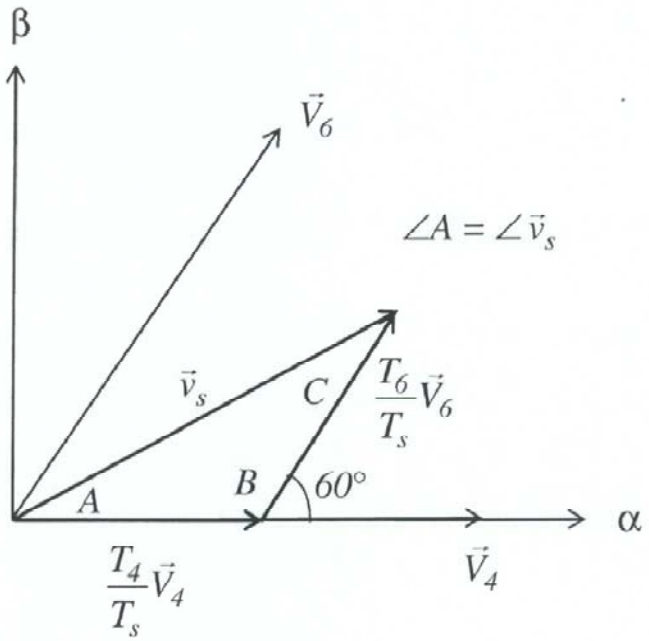


	ρ	θ (°)
$\vec{V}_1[pnn]$	$\sqrt{2} \cdot V_{dc}$	30
$\vec{V}_2[ppn]$		90
$\vec{V}_3[npn]$		150
$\vec{V}_4[npp]$		-150
$\vec{V}_5[nnp]$		-90
$\vec{V}_6[pnp]$		-30
$\vec{V}_0[ppp]$	0	0
$\vec{V}_0[nnn]$		0



$\vec{V}_0 = [ppp] = [nnn]$ at center point





$$\vec{v}_s(t) = \frac{T_0}{T_s}\vec{V}_0 + \frac{T_1}{T_s}\vec{V}_1 + \frac{T_2}{T_s}\vec{V}_2 + \frac{T_3}{T_s}\vec{V}_3 + \frac{T_4}{T_s}\vec{V}_4 + \frac{T_5}{T_s}\vec{V}_5 + \frac{T_6}{T_s}\vec{V}_6 + \frac{T_7}{T_s}\vec{V}_7$$

Note that the desired output voltage vector is in sector 1, inverter voltage vectors V6 (110) and V4 (100) can be

used to produce it. Note also that T_0 to T_7 are the durations of application of voltage vectors V_o to V_7, respectively, for each PWM period T_s.

Comparison of SVM and SPWM

Note that the maximum output voltage vecor $v_{max} = V_{dc}$ when it is along one of the inverter voltage vectors. In between, it is given by the radius of the inscribed circle. To keep the out voltage vector maximum, its tip must fall on the inscribed circle.

Thus, $\bar{v}_{max} = V_{dc}\cos\left(\frac{60°}{2}\right) = 0.866V_{dc}$

$$V_{ph\,max} = \frac{2}{3}\bar{v}_{max} = \frac{V_{dc}}{\sqrt{3}}$$

$$V_{l-l\,max} = \sqrt{3}V_{ph\,max} = V_{dc}$$

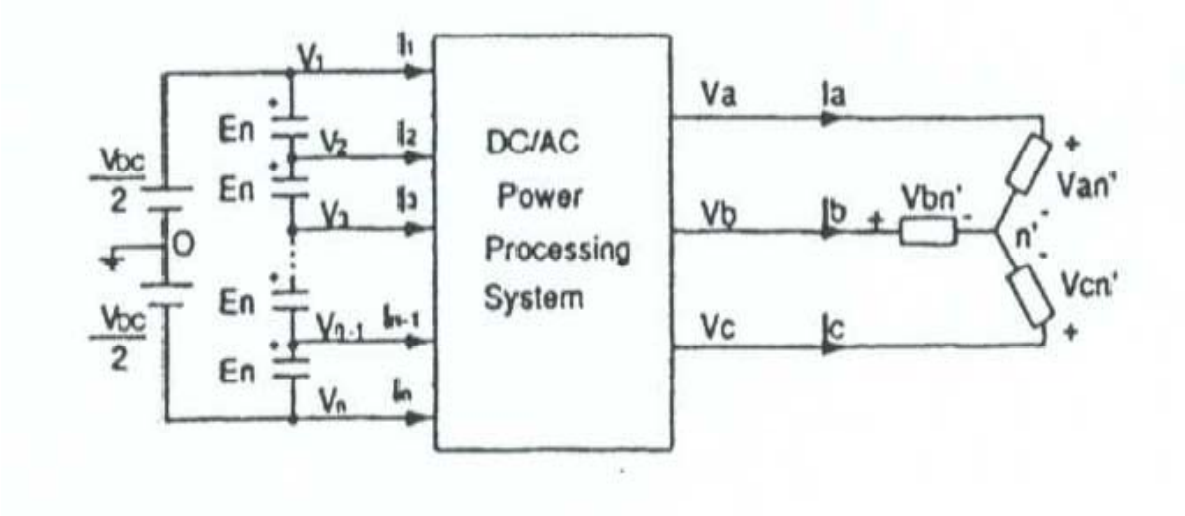
$$V_{l-l\,max\,RMS} = \frac{V_{l-l\,max}}{\sqrt{2}} = 0.707V_{dc}$$

Compared to SPWM, for SVM is

$V_{l-l\,max\,RMS} = \frac{0.707}{0.612} = 15.5\%$ more than obtained with SPWM.

SVM can also include overmodulation, in which case, the maxumum output voltage vector traces the outer hexagon.

Multi-level Inverters

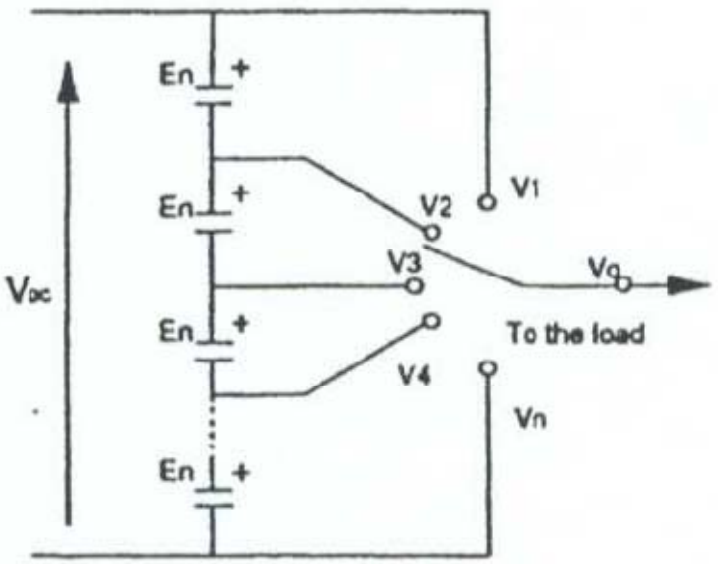


1. Mainly for applications at high voltage (power).

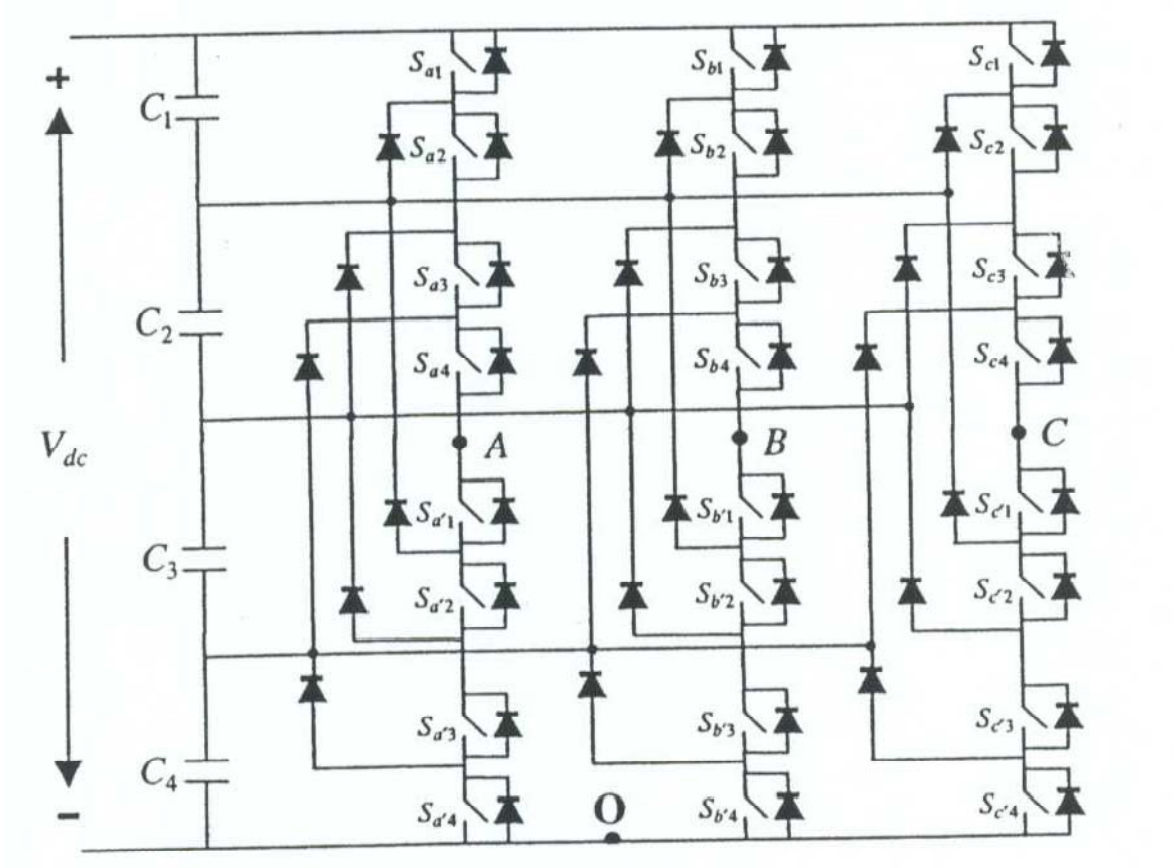
2. High voltage stresses of 2-level inverters are avoided

3. Lower switching frequencies are used with high THD performance.

4. Reduced EMI and voltage reflection problems



Diode-Clamped ML Inverter

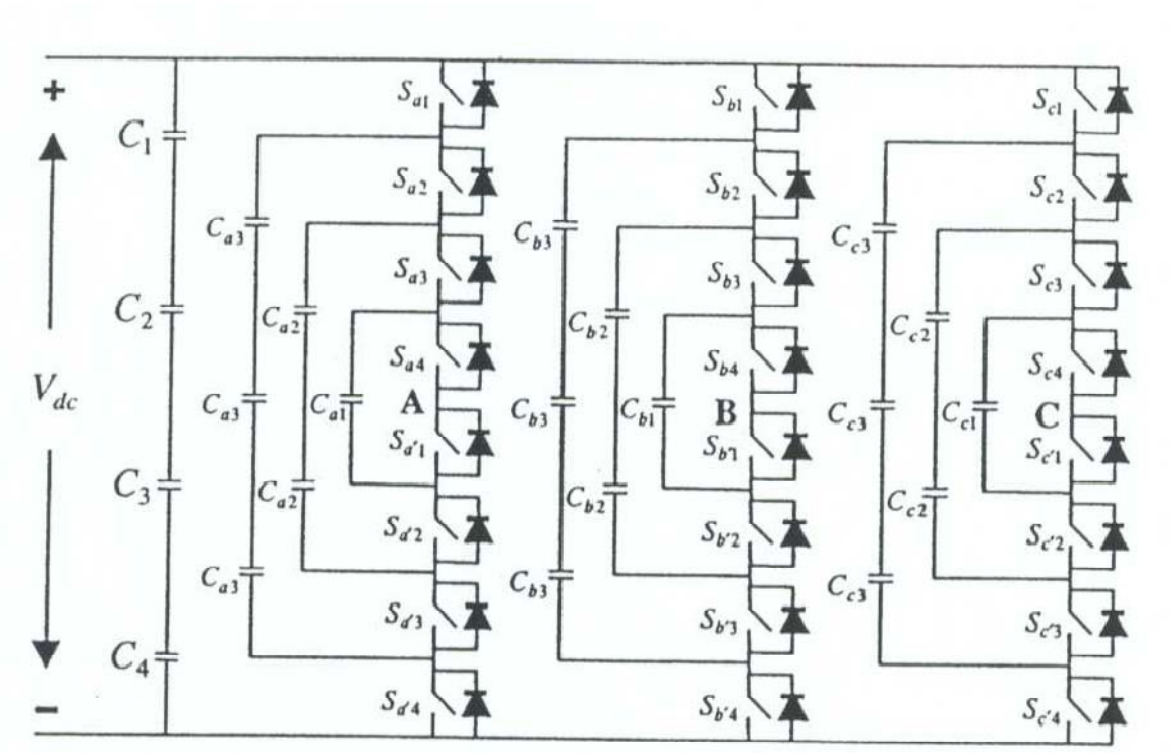


Output V_{AO}	Switch State							
	S_{a1}	S_{a2}	S_{a3}	S_{a4}	$S_{a'1}$	$S_{a'2}$	$S_{a'3}$	$S_{a'4}$
$V_5=V_{dc}$	1	1	1	1	0	0	0	0
$V_4=3Vdc/4$	0	1	1	1	1	0	0	0
$V_3=Vdc/2$	0	0	1	1	1	1	0	0
$V_2=Vdc/4$	0	0	0	1	1	1	1	0
$V_1=0$	0	0	0	0	1	1	1	1

Voltage levels and switch states of a 5-level inverter

Note that each switch now blocks only $V_{dc}/4$ volts! Also all switches are only switched at the fundamental frequency!

Flying Capacitor ML Inverter

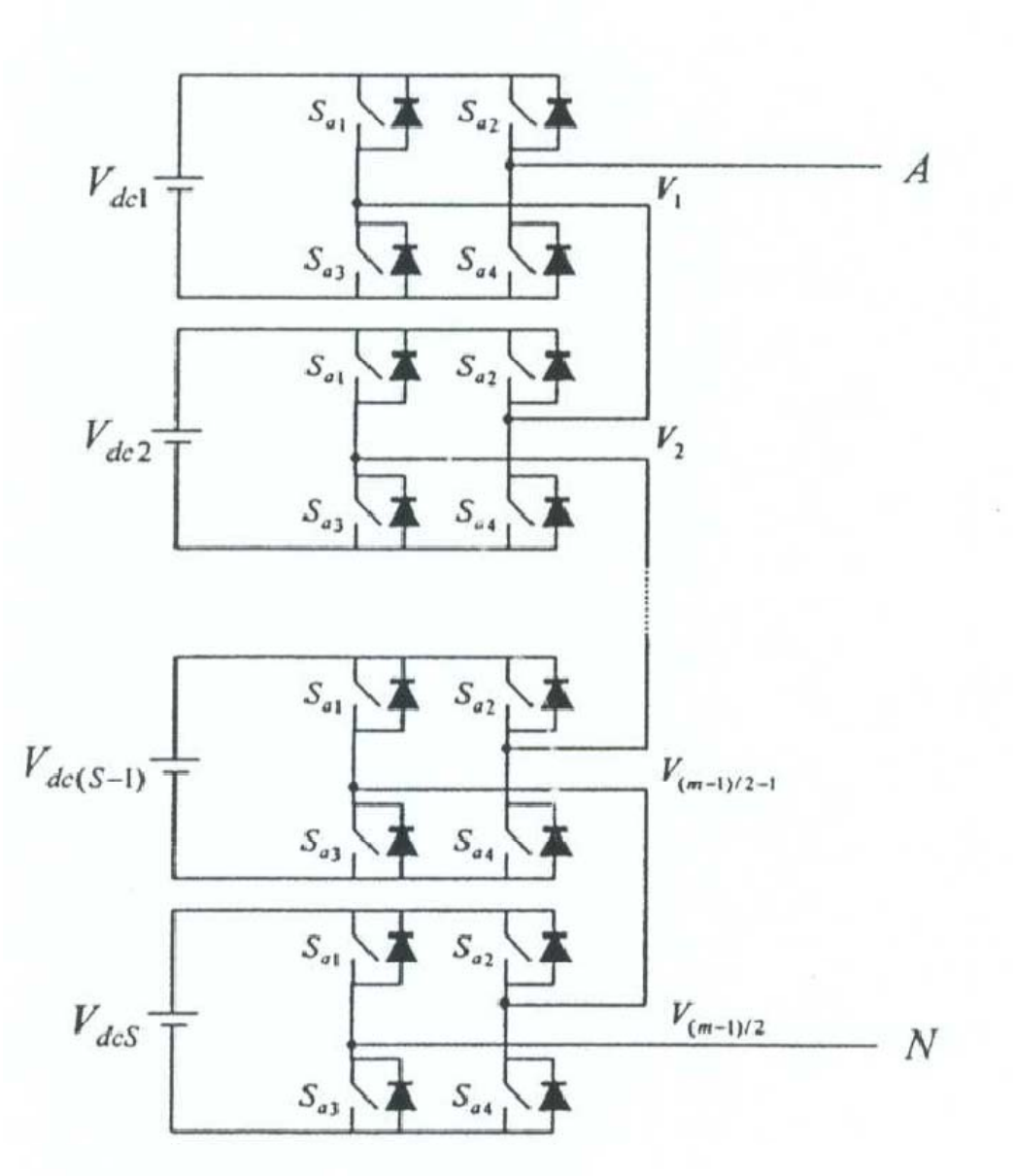


Output V_{AO}	Switch State							
	S_{a1}	S_{a2}	S_{am-1}	S_{am}	$S_{a'1}$	$S_{a'2}$	$S_{a'm-1}$	$S_{a'm}$
$V_5=V_{dc}$	1	1	1	1	0	0	0	0
$V_4=3V_{dc}/4$	1	1	1	0	1	0	0	0
$V_3=V_{dc}/2$	1	1	0	0	1	1	0	0
$V_2=V_{dc}/4$	1	0	0	0	1	1	1	0
$V_1=0$	0	0	0	0	1	1	1	1

Switching states

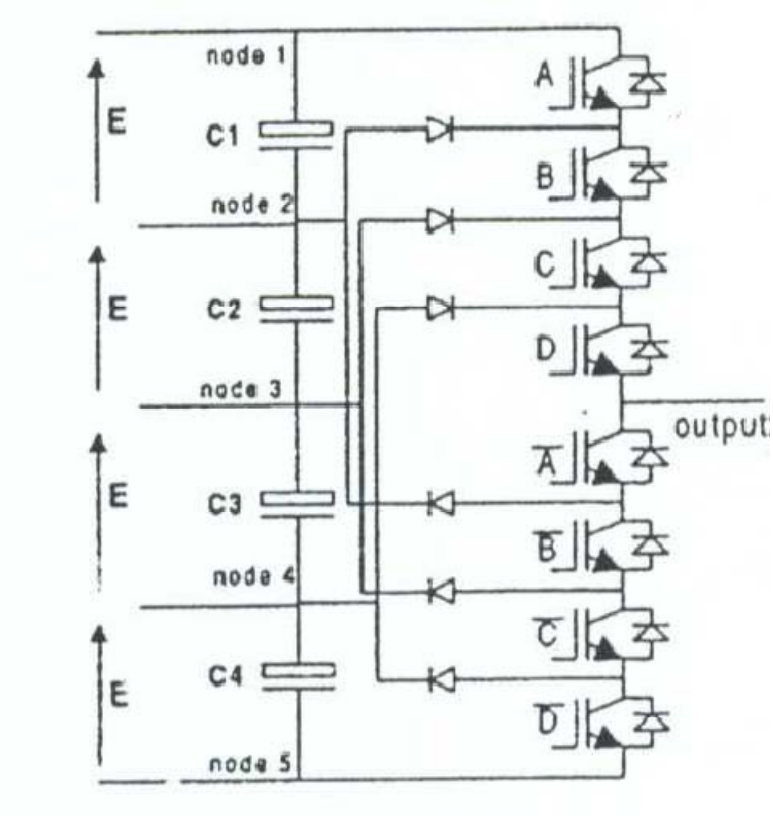
Each capacitor voltage is different from the next. The size of voltage increments between capacitors determine the levels of output voltage.

Cascaded ML Inverter



Several isolated DC supplies are used.

5-level ML Inverter



State	A	B	C	D	Output voltage
1	0	0	0	0	-2E
2	0	0	0	1	-E
3	0	0	1	1	0
4	0	1	1	1	+E
5	1	1	1	1	+2E

Table 4.1 Switching states

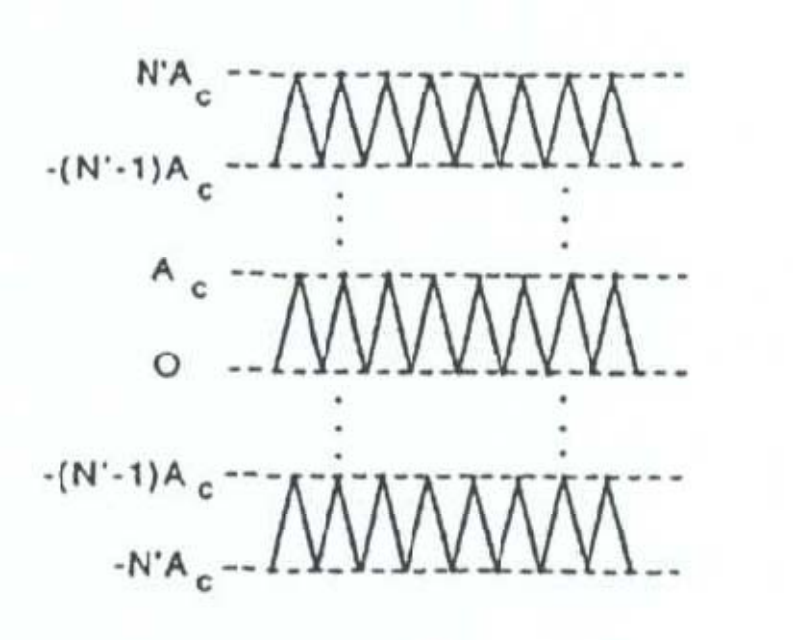
Control strategy

SPWM is used

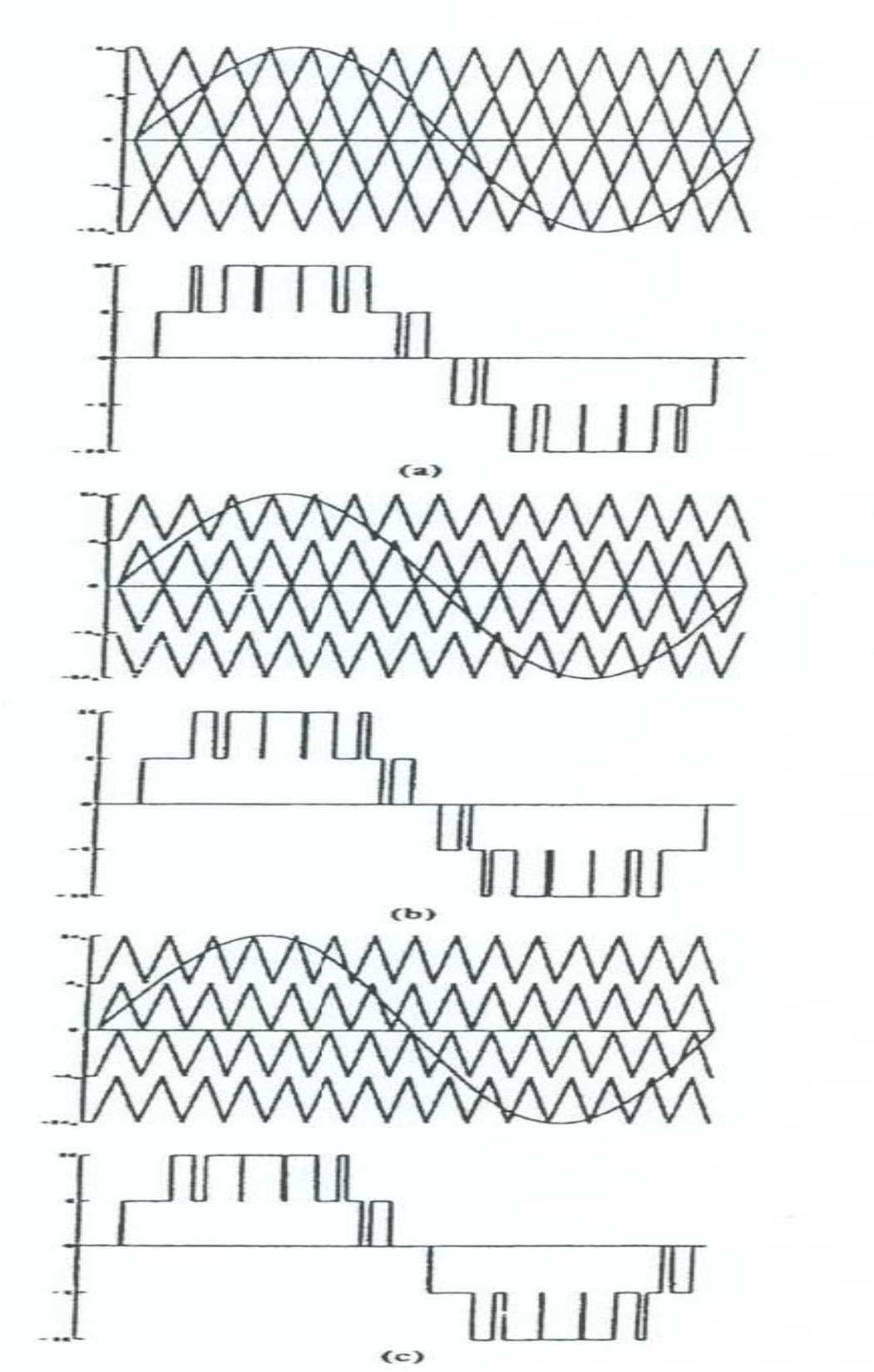
$$m_f = \frac{f_c}{f_m}$$

$$m_a = \frac{A_m}{(N'.A_c)}$$

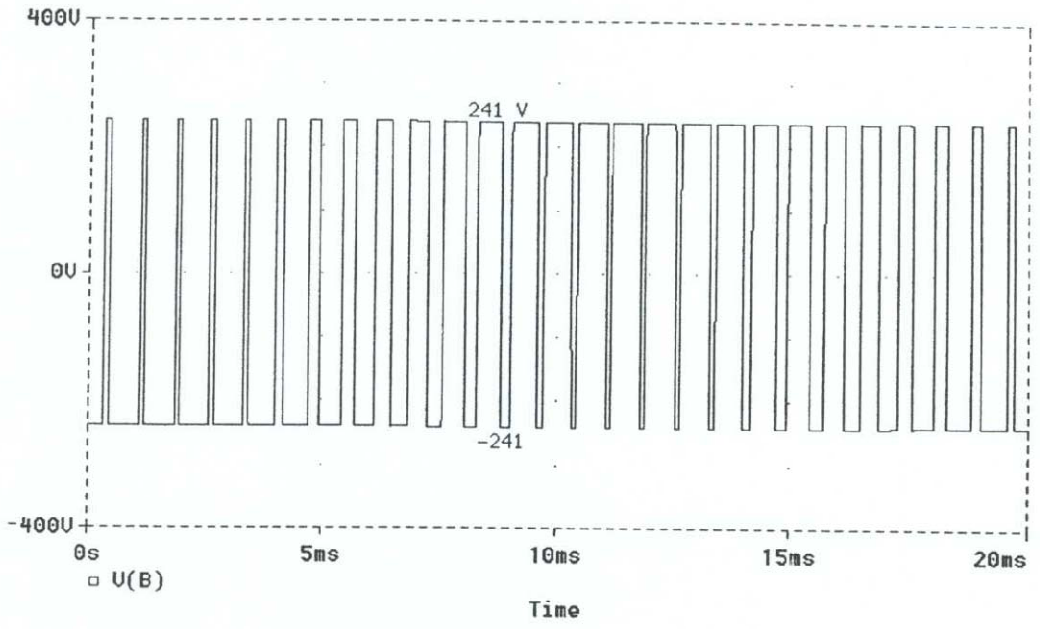
- A_m is the amplitude of the modulating (reference) signal
- A_c is the amplitude of the carriers
- $N' = (N-1)/2$ where N is the level of the inverter, say 5.



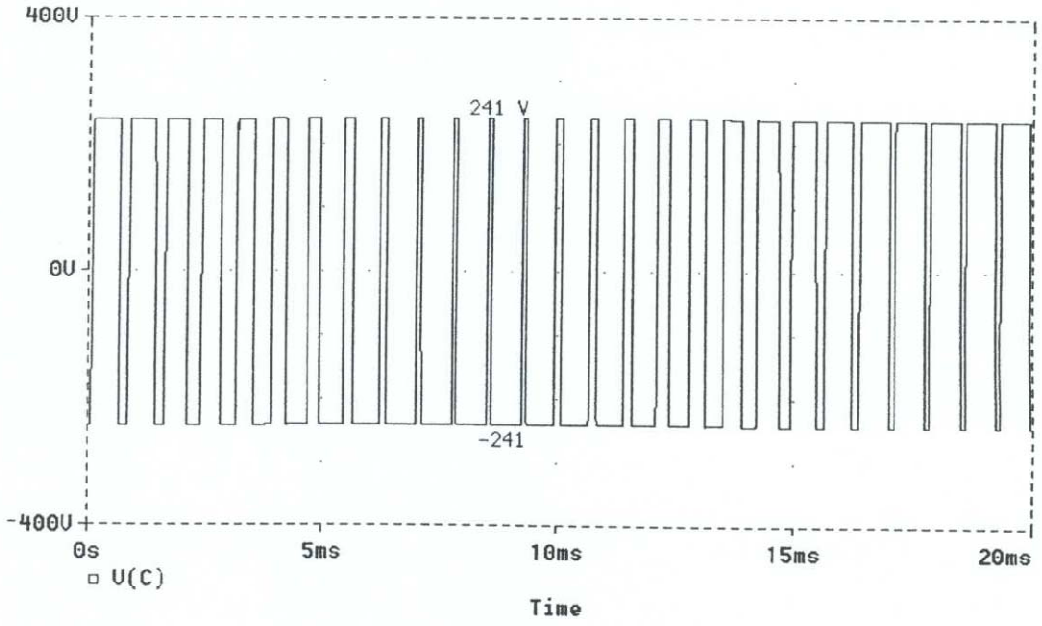
Carrier decomposition



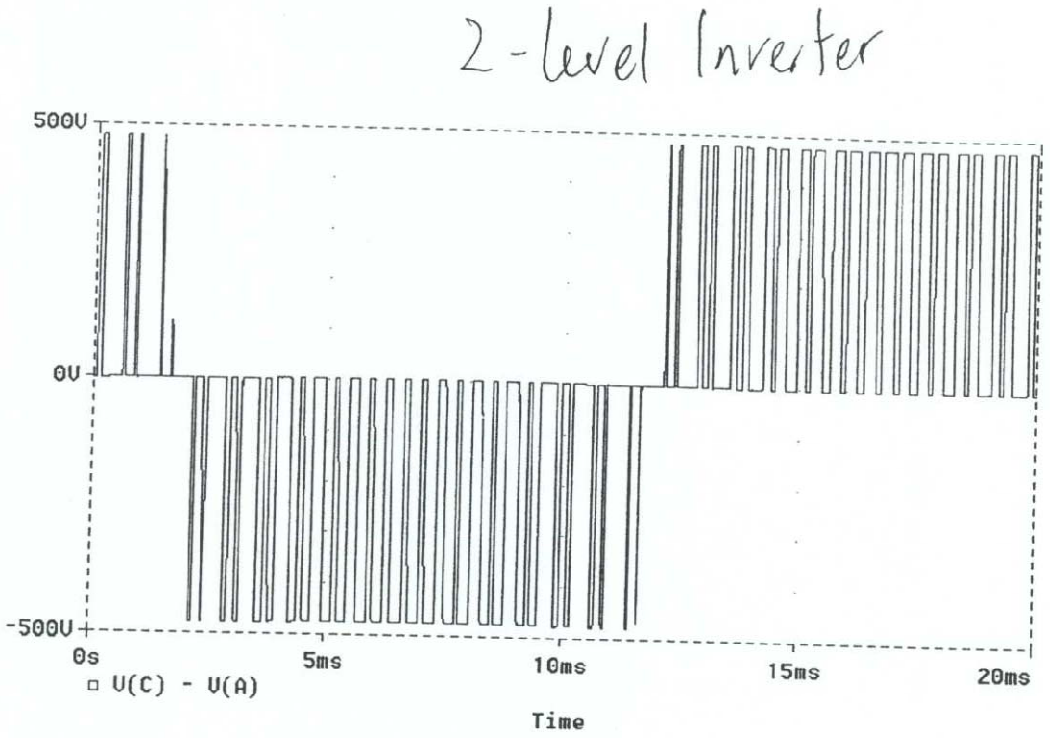
2-level Inverter



(b) Voltage waveform at pole-B

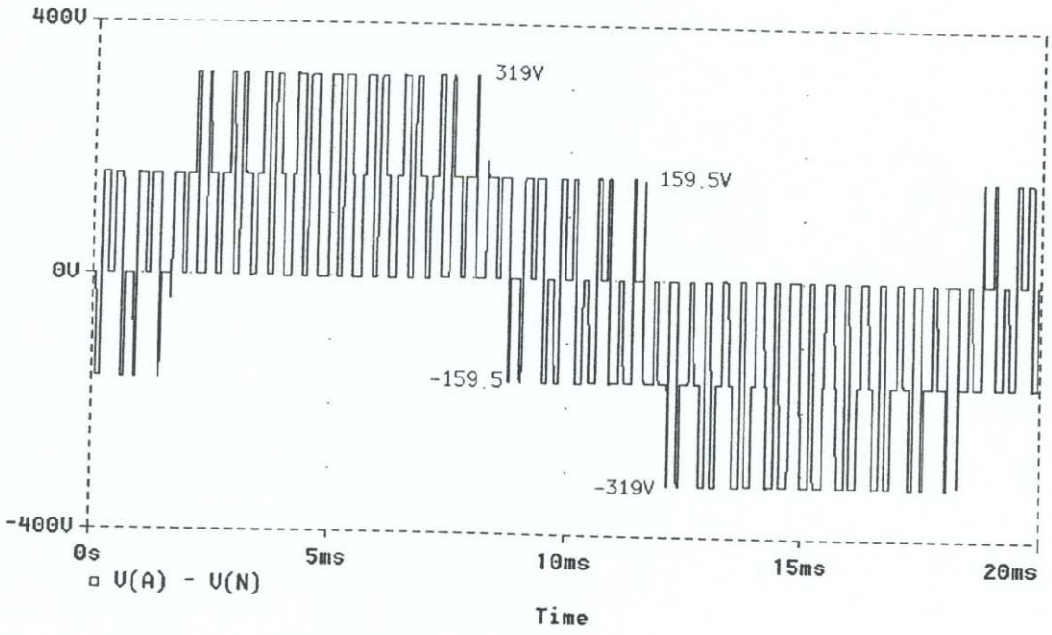


(c) Voltage waveform at pole-C



(c) Line-to-line waveform: V_{CA}

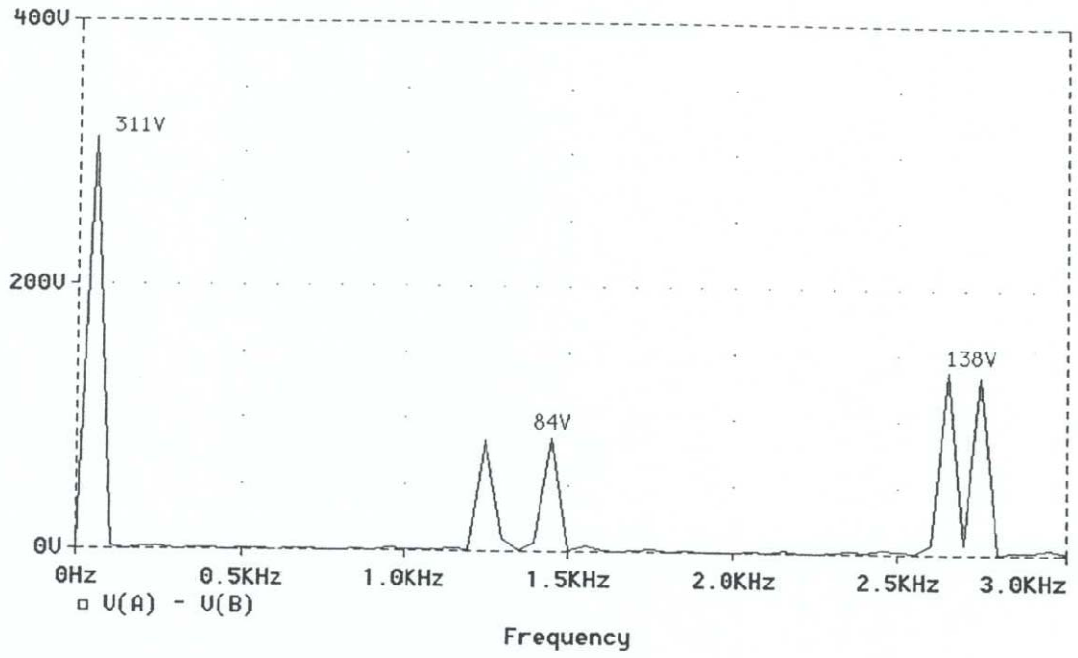
Fig. 6.6 Line-to-line voltage waveforms: (a) V_{AB}, (b) V_{BC}, and (c) V_{CA}



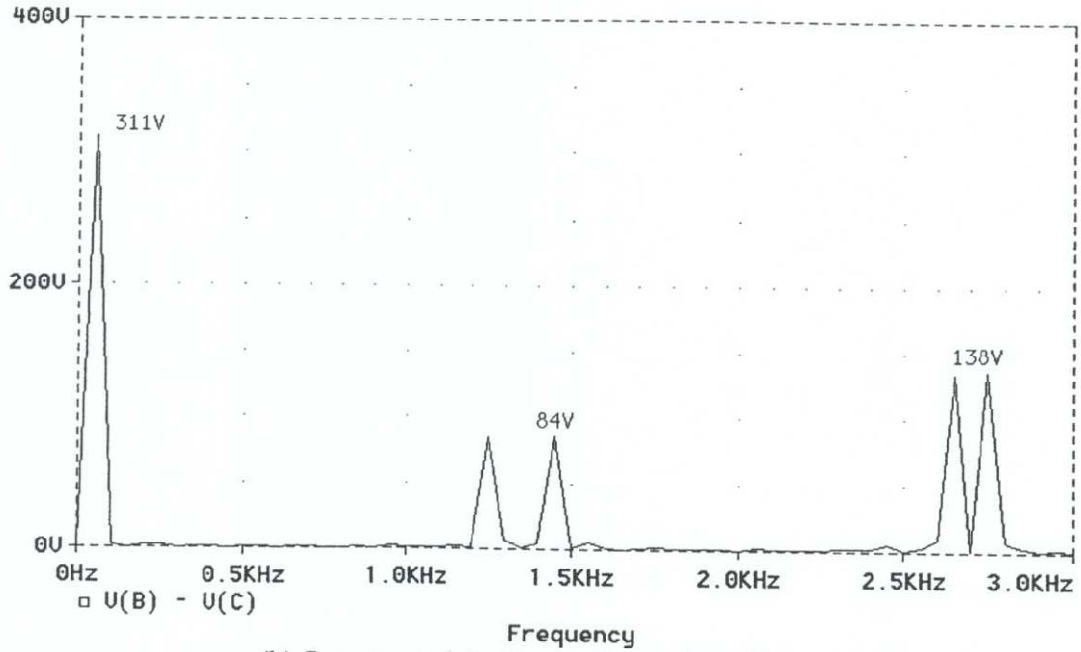
(a) Line-to-neutral waveform: V_{AN}

2-Level Inverter

6.1.3 Spectra of the Voltage Waveforms

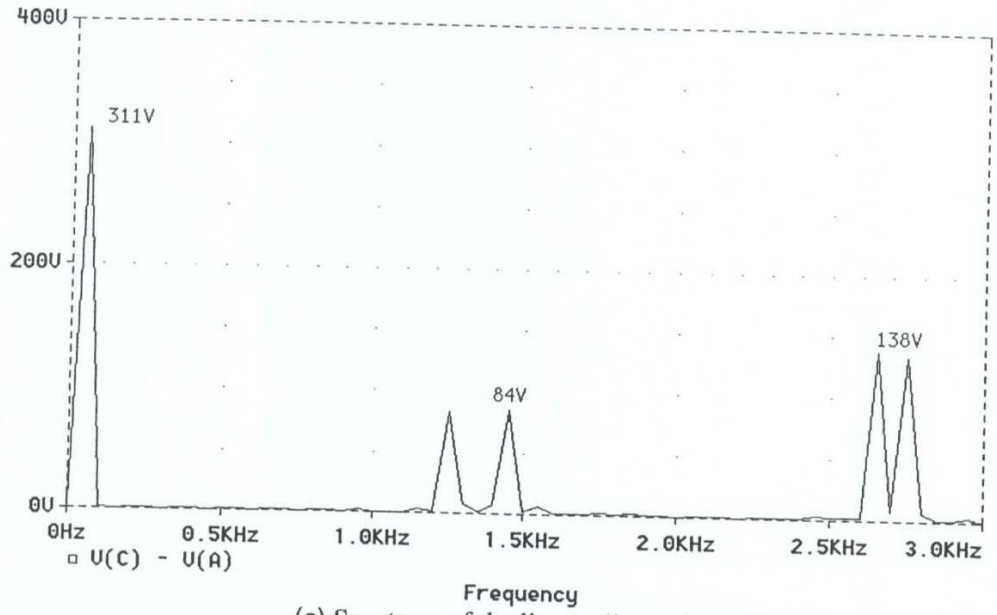


(a) Spectrum of the line-to-line voltage V_{AB}



(b) Spectrum of the line-to-line voltage V_{BC}

2-level Inverter



(c) Spectrum of the line-to-line voltage V_{CA}

Fig. 6.8 Harmonic spectrum of the line-to-line voltage: (a) V_{AB}, (b) V_{BC}, and (c) V_{CA}

6.1.4 Line Current Waveforms

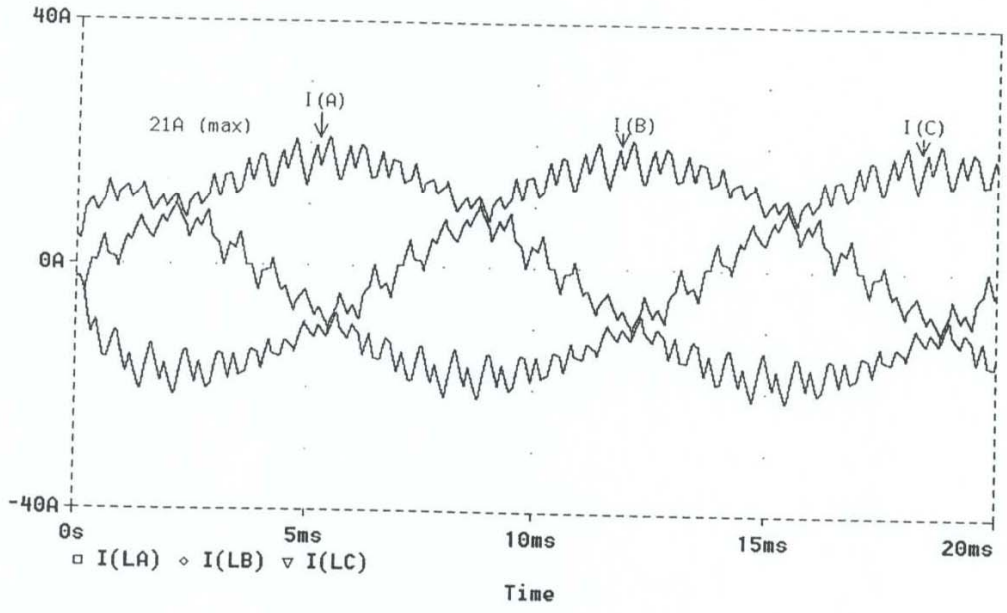
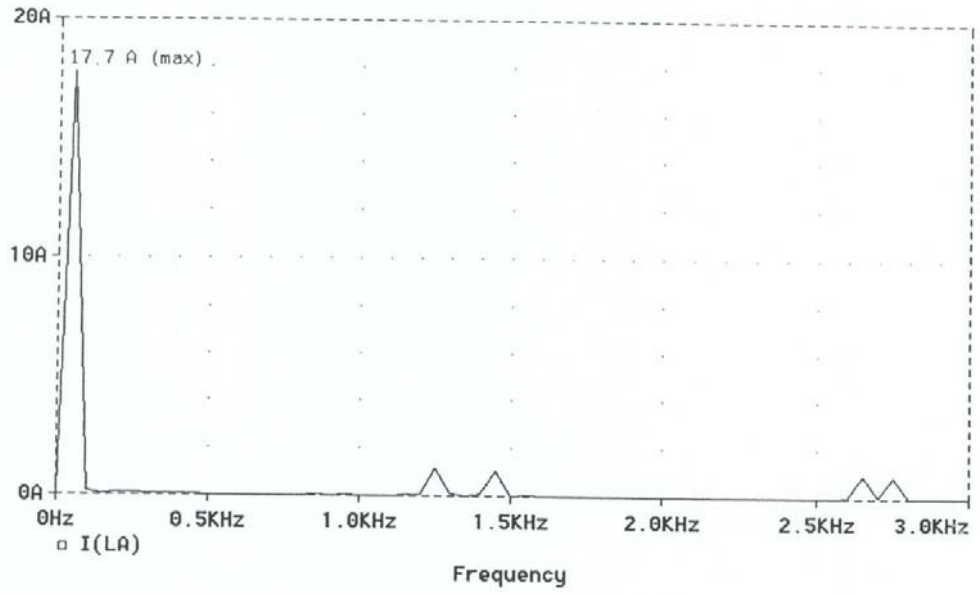


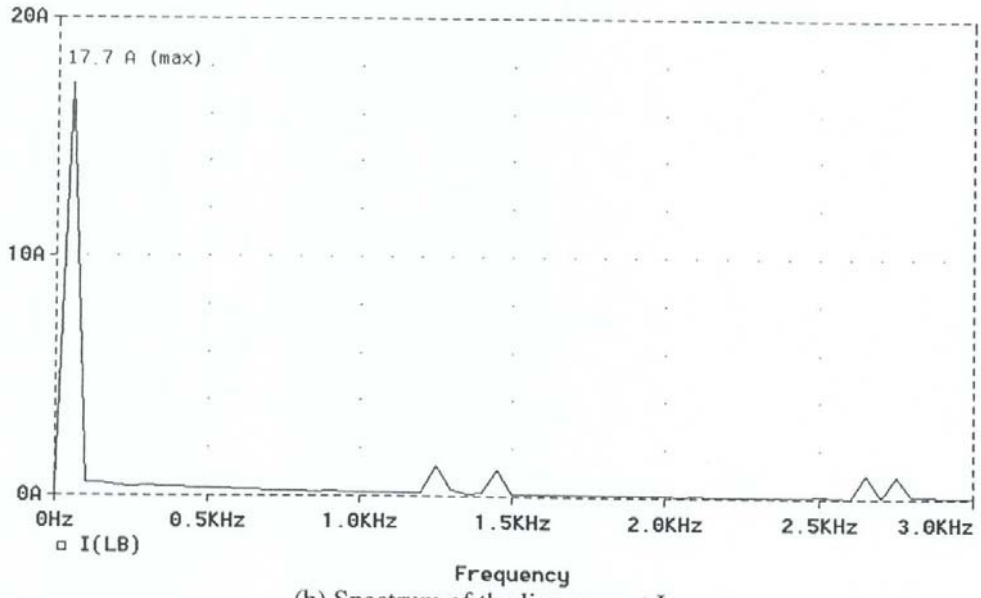
Fig. 6.9 Line current waveforms for two-level inverter

2-level Inverter

6.1.5 Spectra of the Line Currents

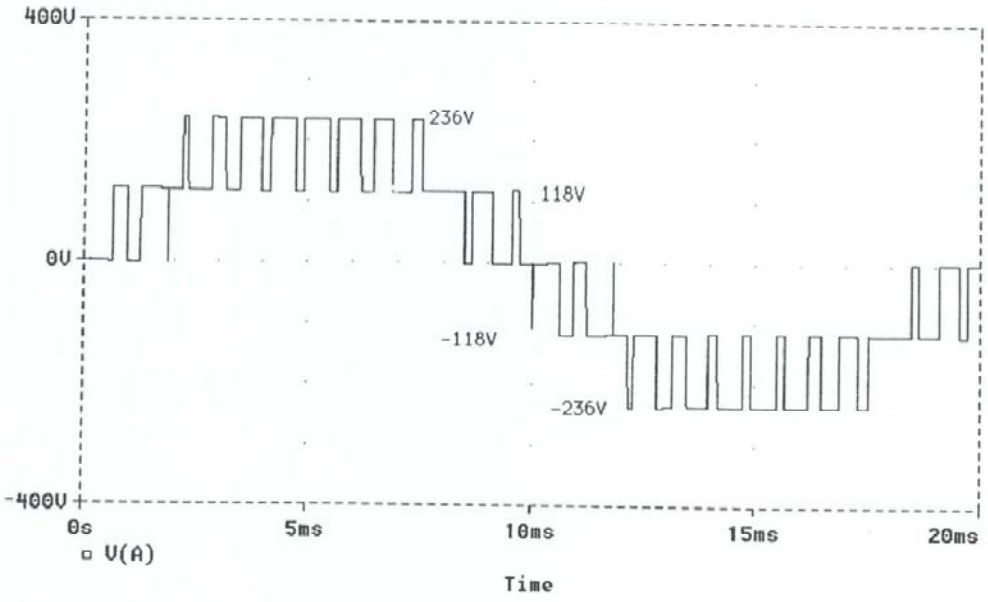


(a) Spectrum of the line current I_A

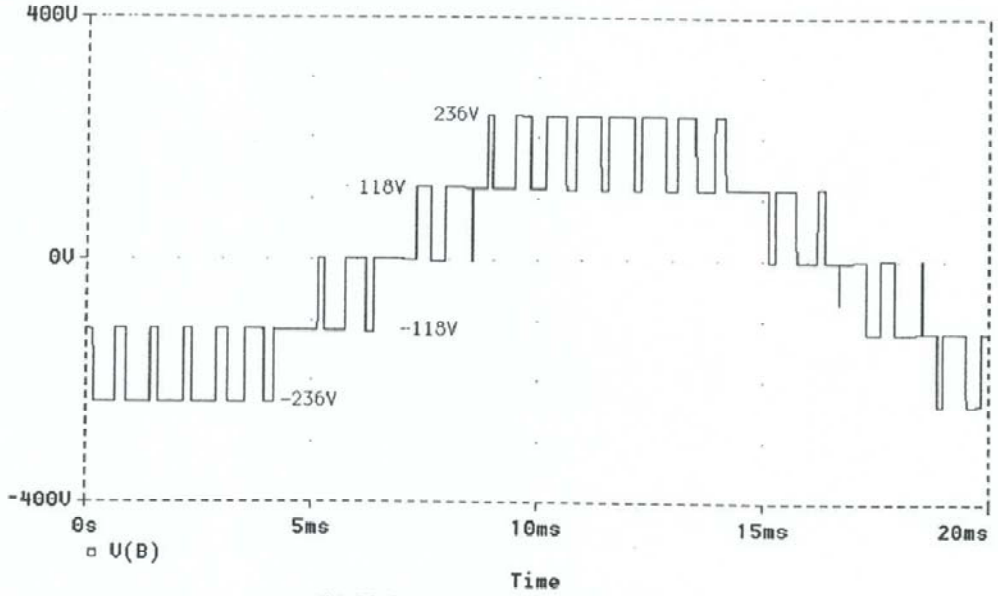


(b) Spectrum of the line current I_B

5-level Inverter

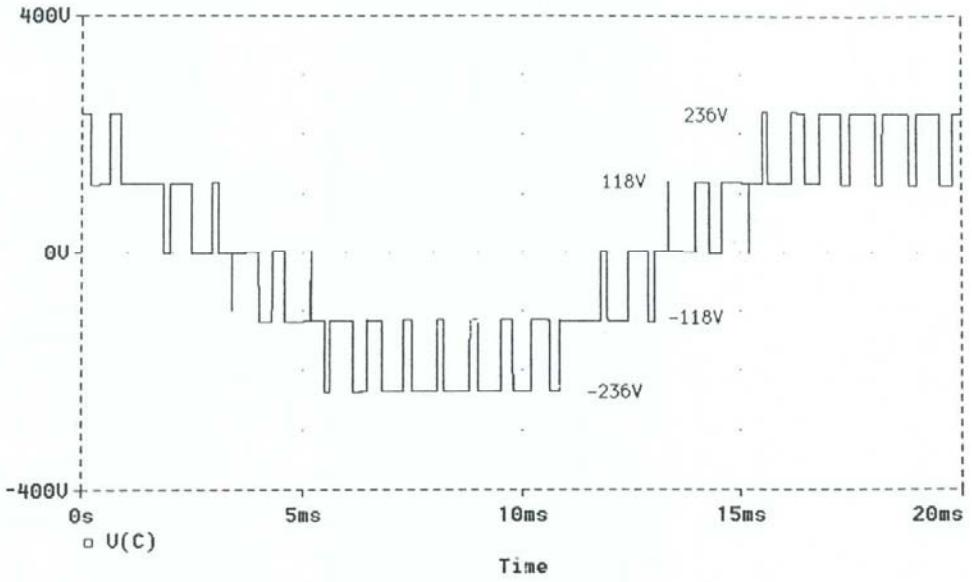


(a) Voltage waveform at pole A

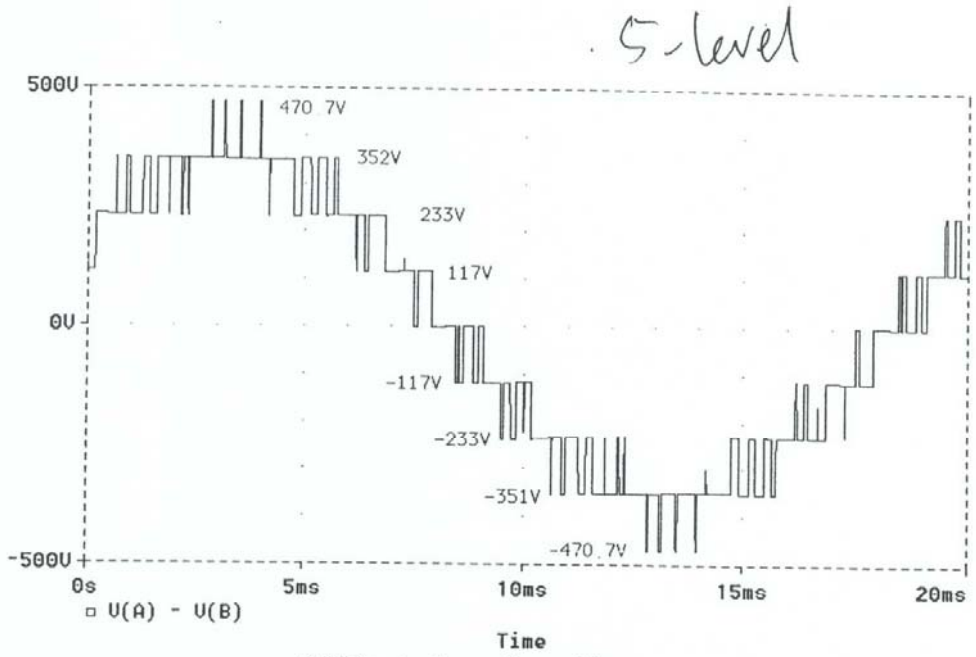


(b) Voltage waveform at pole B

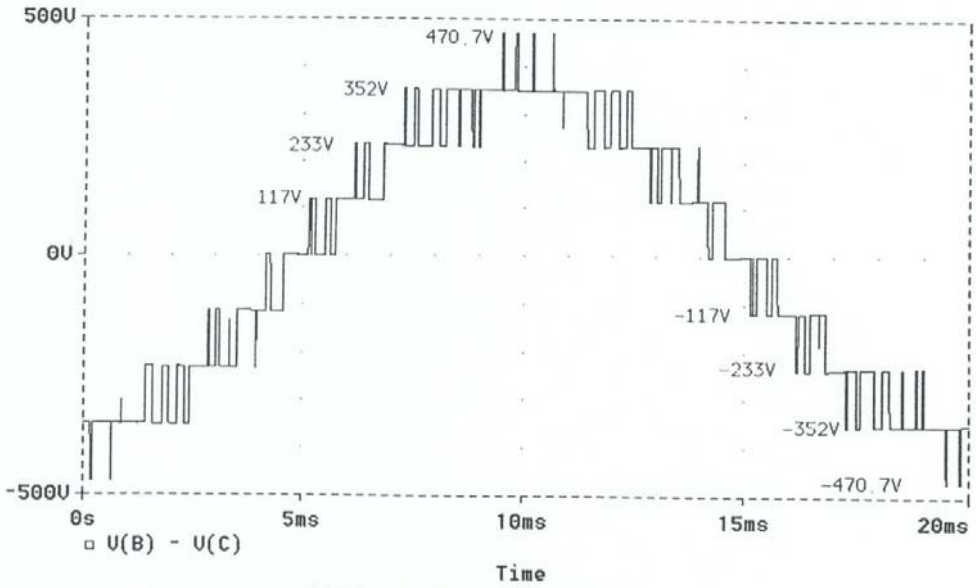
5-Level Inverter



(c) Voltage waveform at pole C

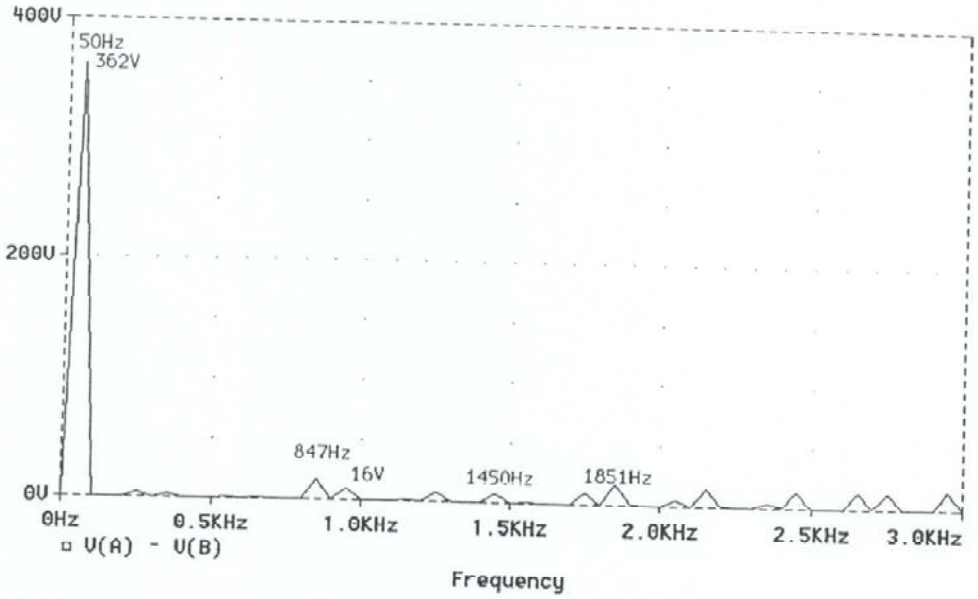


(a) Line-to-line voltage: V_{AB}

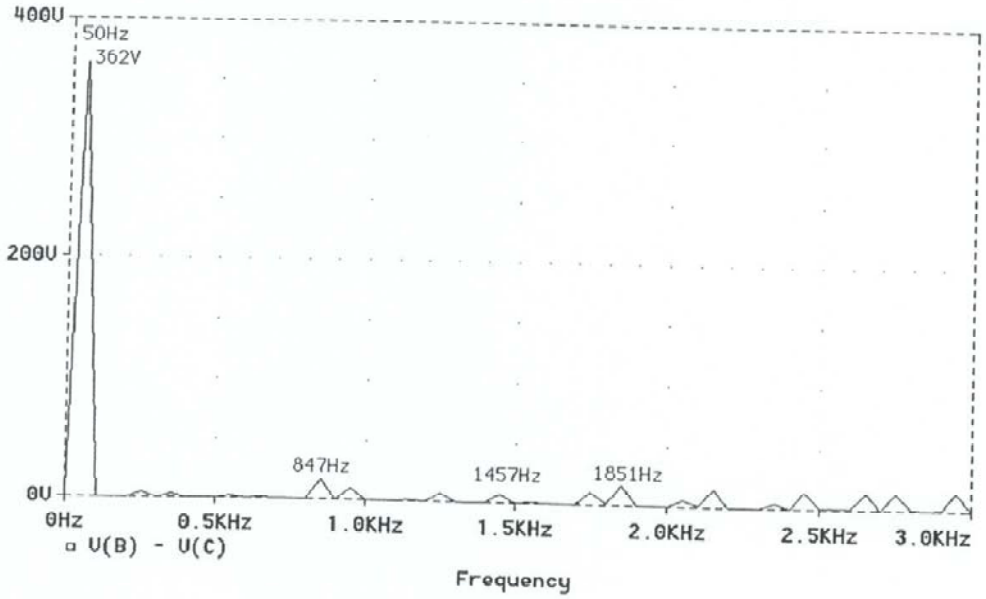


(b) Line-to-line voltage: V_{BC}

5- level



(a) Spectrum of the line-to-line voltage V_{AB}



(b) Spectrum of the line-to-line voltage V_{BC}

Line-line voltage spectra

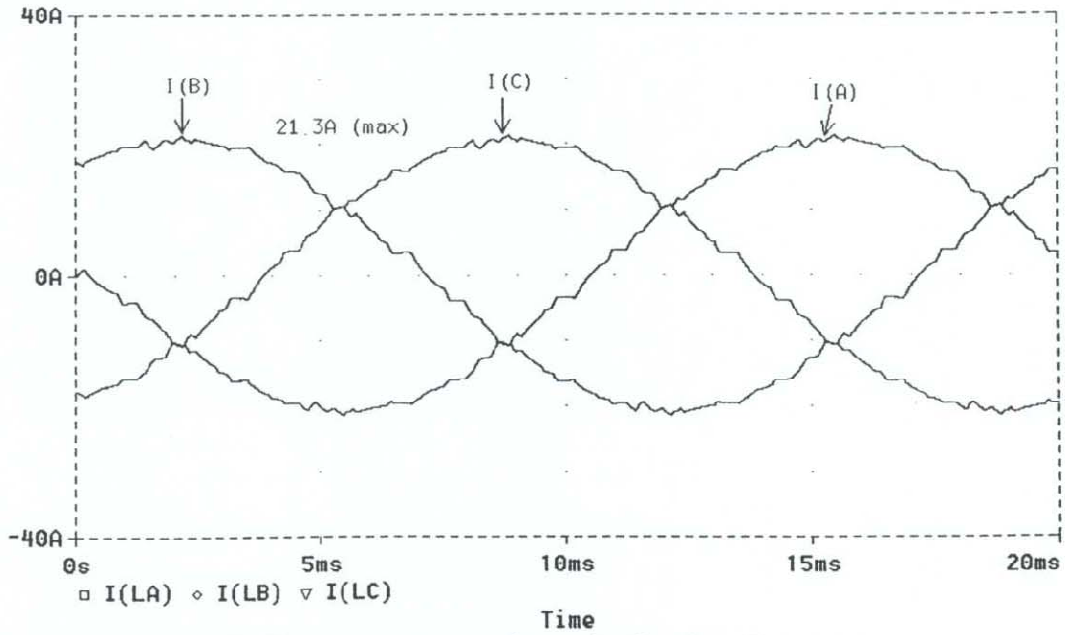
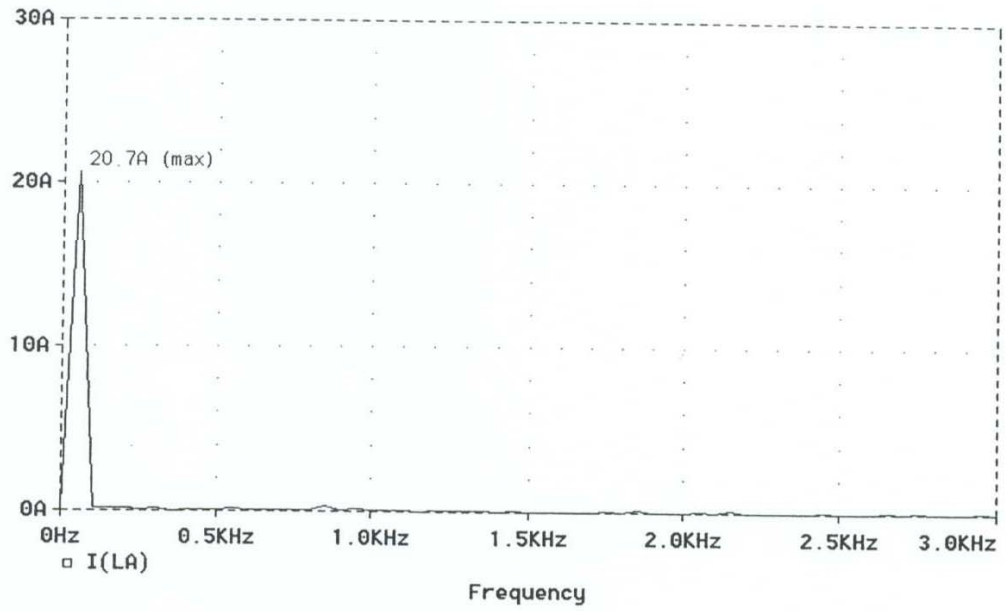


Fig.6.27 Line current waveforms for five-level inverter

5-level

6.2.5 Spectra of the Line Currents



(a) Spectrum of the line current I_A